Simcenter Flotherm™ BCI-ROM Validation

Software Version 2019.1

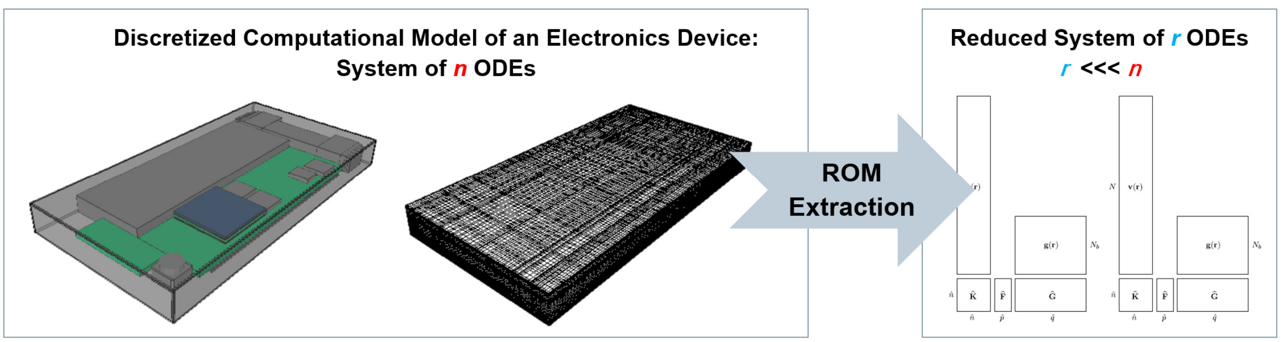
June 2019

1.0 Introduction

Modern electronics design requires consideration of transient power loads from mission profiles or power control strategies, while the product is situated in a wide range of thermal environments. The computational time required to solve a full CFD model is often prohibitive, preventing the full transient design space from being explored.

To address this problem, Simcenter Flotherm 2019.1 introduces the capability to extract a Boundary Condition Independent (BCI) Reduced Order Model (ROM) from a meshed conduction model.

Reduced Order Modeling is an alternative approach to extract a dynamic compact thermal model (DCTM) from a thermal simulation model. A discretized computational model of an electronics device is a system of ***n*** simultaneous linear equations that must be solved. The ROM extraction process aims to derive a reduced system of ***r*** equations that will produce thermal results in space and time within a specified tolerance of the original set of ***n*** equations. If ***r << n***, the computational resources and time required to solve the equations is drastically reduced.



A model is BCI if it predicts temperature in space and time with accuracy regardless of the thermal environment in which it is placed. A BCI-ROM will enable extremely fast transient solutions while retaining high accuracy in all situations.

The BCI-ROM extraction method in Simcenter Flotherm is an extension of the FANTASTIC method. The literature[1,2,3,4] generally describes the method and presents several validation examples.

The accuracy of the BCI-ROM extracted with this method can be bounded mathematically by the tolerance (ϵ) used for extraction. Specifically:

- The Hankel norm of the error of the impedance matrix < 2ϵ
- The energy norm of the error of the space-time distribution of temperature rises in response to power impulses < $2\sqrt{\epsilon}$

The reader is encouraged to review the literature for an explanation of these bounds.

This document will expand on the validation examples in the literature, and demonstrate the use of the BCI-ROM method for several typical electronics applications. The results will be presented in the form of maximum percent error between Simcenter Flotherm and BCI-ROM results, a quantity more familiar than the formal mathematical bounds described above.

2.0 Functional Overview

Use of the BCI-ROM extraction feature has several pre-requisites. The Simcenter Flotherm model:

- Must be conduction only. Regions of the model that are fluid are considered as solid with the properties of the primary fluid in the model.
- Must be linear. Temperature dependent material properties, radiative exchange, joule heating, etc. are not supported.
- Must have at least one non-zero heat transfer coefficient (HTC) defined in an attached Ambient attribute.
- Must have at least one volume heat source defined with a total power setting. This can either be a Source attribute or a Thermal attribute.

The BCI-ROM extraction solver is controlled with two settings:

- A numerical tolerance (acceptable relative error): ϵ
- A heat transfer coefficient (HTC) range.

The tolerance is somewhat akin to the termination residual in the Simcenter Flotherm solver. It describes the acceptable relative difference between the full CFD results and the ROM results in space and time[1,2,3,4]. The valid range for this parameter is $0.0001 \leq \epsilon \leq 0.01$. The default value of $\epsilon = 0.001$ is recommended in most situations. The results in this document will establish the motivation for this default value and the allowable range.

The HTC range sets the extent of the BCI scope for the ROM. If the HTC range is set as 1 – 10,000 W/m^{2}K, then the ROM is ensured to be BCI within that range only.

The BCI-ROM extraction is an iterative calculation. The time and memory required to complete is generally not predictable in advance, but will increase as:

- the number of heat sources increases
- the number of attached ambient attributes increases
- the extraction HTC range increases
- the extraction tolerance decreases
- the number of grid cells in the model increases

Simcenter Flotherm can convert the BCI-ROM into two different formats for downstream analysis:

- Matrices compatible with Matlab and the open source tool, Octave.
- Thermal netlist file (.sp extension) compatible with Mentor Eldo.

The matrices format is BCI and will be used for all of the validation work in this document. The thermal netlist format is not BCI, and is valid only for the set of HTCs defined in the original model.

3.0 Validation Approach

For each model in Section 4, the following procedure was followed to quantify the accuracy of the ROM and to confirm BCI.

- Preliminary: The Simcenter Flotherm model was configured such that all heat sources are powered on at t=0 and remained constant throughout the simulation. The transient duration must be sufficient to ensure the system reaches a steady state. This step change of power will exercise the model at all frequencies, hence it's desirability as a test case.
- Select an extraction HTC range and an extraction tolerance.
- Using Design of Experiments (DoE) in Command Center, create and solve many variants of the Flotherm model, each with a unique combination of HTC values within the extraction HTC range. Record temperature vs time for each Monitor Point.

- Extract the BCI-ROM and solve it using the same variants of HTCs used in Command Center. Ensure that the same time grid is utilized for the ROM solve. Record temperature vs time for each BCI-ROM Monitor Point.
- For each HTC scenario, compare the Simcenter Flotherm temperature to the BCI-ROM temperature at every time step and determine the maximum percent error.
- Considering all HTC scenarios, determine and record the maximum, mean, and standard deviation of the maximum errors.
- Repeat the steps above for several HTC ranges and several tolerances. The values used in this work are as follows:
    - HTC Ranges:
        - 1-100 W/m^2K
        - 1-1000 W/m^2K
        - 1-10,000 W/m^2K
        - 1-100,000 W/m^2K
        - 1-1,000,000 W/m^2K
    - Extraction Tolerances:
        - 0.1, 0.05, 0.02, 0.01, 0.005, 0.002, 0.001, 0.0005, 0.0002, 0.0001

4.0 Validation Cases

4.1 Package on Package

4.1.1 Description of Model

The model represents a Package on Package (PoP) device, consisting of 2 stacked wire bonded BGA packages. Each of the packages is modelled with a single heat source on the die.

There are three ambient attributes defined and attached to the various faces of the model. In the original model:

- The ambient attributes are defined as:
    - Top: HTC = 20 W/m^{2}K and $T_{ambient}$ = 0 degC.
    - Sides: HTC = 500 W/m^{2}K and $T_{ambient}$ = 0 degC.
    - Bottom: HTC = 200 W/m^{2}K and $T_{ambient}$ = 0 degC.

- The heat sources are defined as:
    - Top Die: 2 W
    - Bottom Die: 3 W

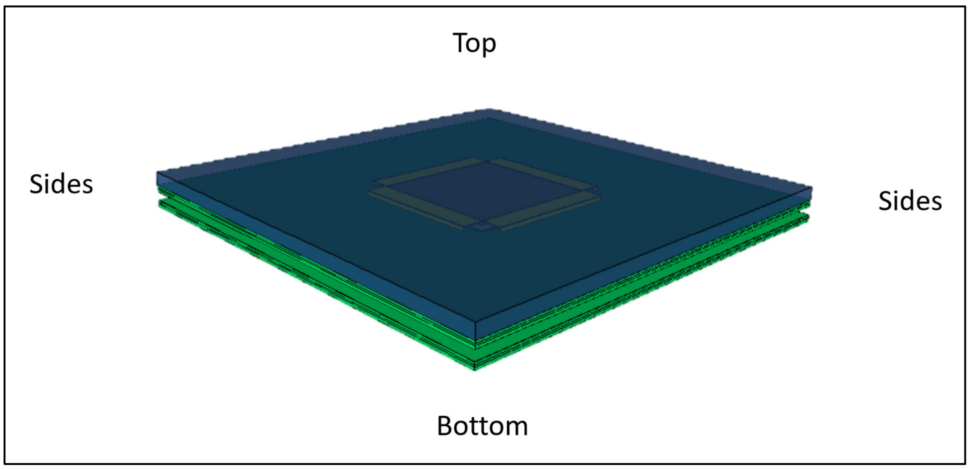


Figure 1: Package on Package model. The labels approximately indicate the location of the attached ambients.

The model contains 150,210 grid cells. For validation purposes, both sources were turned on at t=0, and the transient duration set to 700s.

4.1.2 Model Variations

Command Center is used to create 40 scenarios with different combinations of HTC sets (using Design of Experiments). This was repeated for the following HTC ranges:

- HTC Range:
    - 1-100 W/m^{2}K
    - 1-1000 W/m^{2}K
    - 1-10,000 W/m^{2}K
    - 1-100,000 W/m^{2}K
    - 1-1,000,000 W/m^{2}K

Figure 2 is a chart showing the HTC distributions for the HTC range of 1 to 10,000 W/m²K. The distributions for the other ranges have a similar pattern.

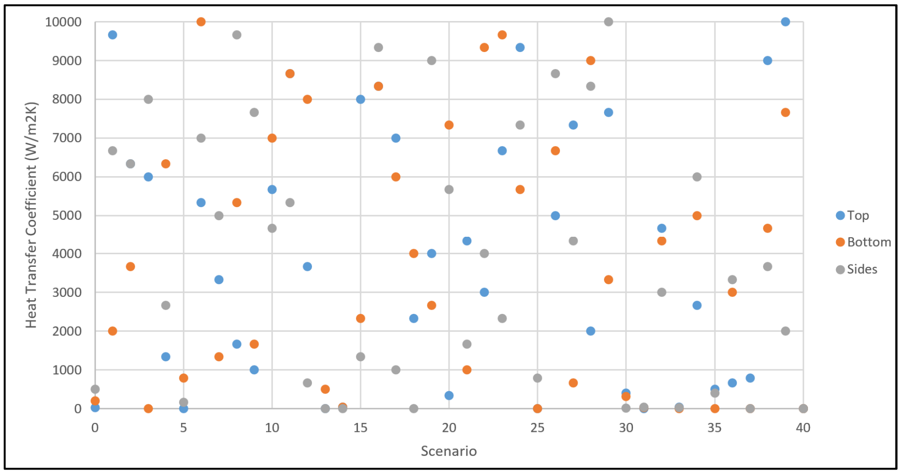


Figure 2 – Combinations of HTC tested

4.1.3 Results: HTC Range: 1-10,000 and ϵ = 0.001

As described in Section 3, the full temperature response for each heat source monitor point is compared for every time step. The comparison for a single scenario is shown in Figure 3. The curves are nearly identical at all time steps. The maximum percent error for this scenario is 0.0737%.

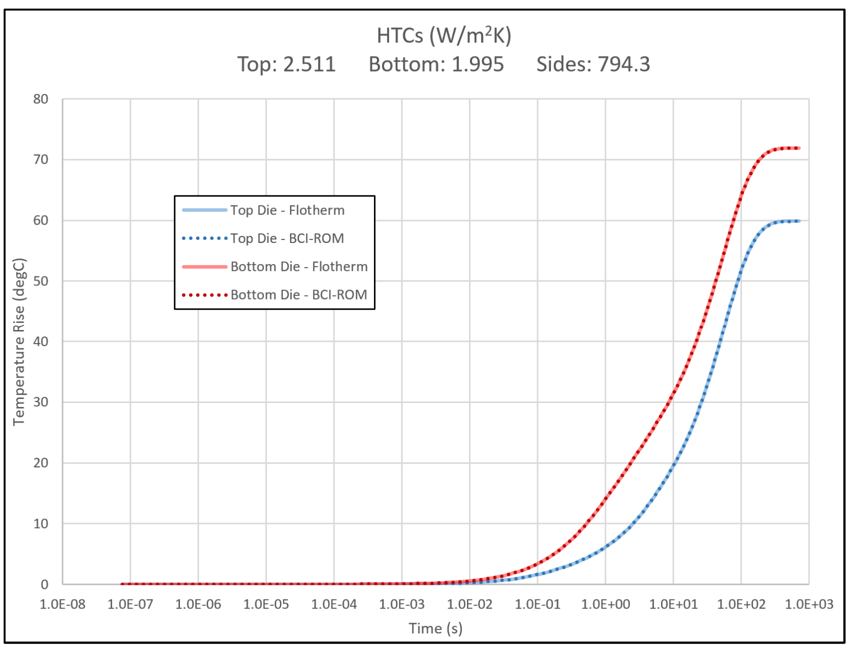


Figure 3 – Maximum Percent Error = 0.0737%

Figure 4 reports the maximum percent error for each scenario tested. The maximum error encountered considering all scenarios was 0.202%. This demonstrates the BCI nature of the ROM.

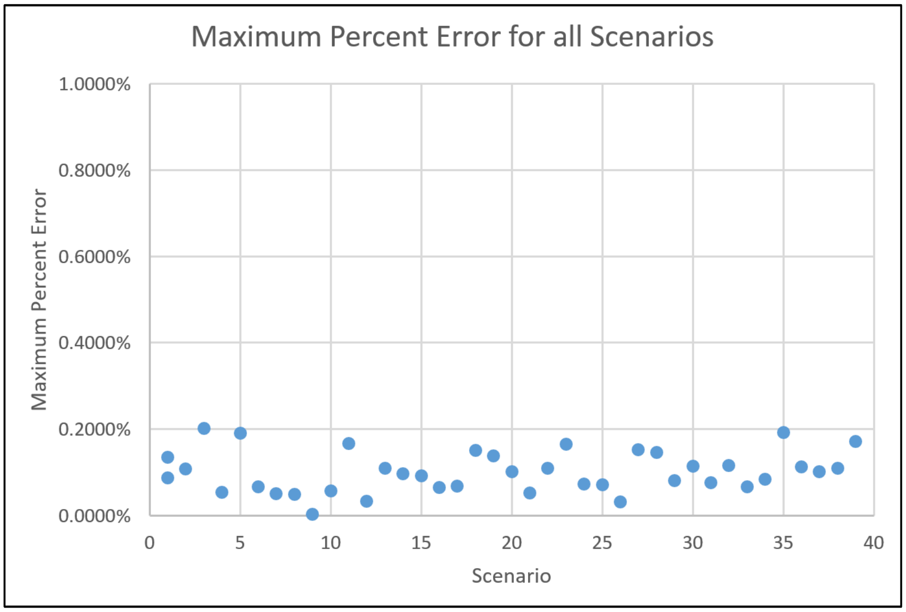


Figure 4 – Maximum Percent Errors for all Scenarios.
HTC Range = 1 – 10,000 W/m²K and ϵ = 0.001
Max = 0.202%, Mean = 0.101%, Standard Deviation = 0.0165%

4.1.4 Impact of Extraction Tolerance, ϵ

The impact of the extraction tolerance, ϵ, on BCI-ROM accuracy and creation time for this model for various HTC ranges is shown in Figures 5-9.

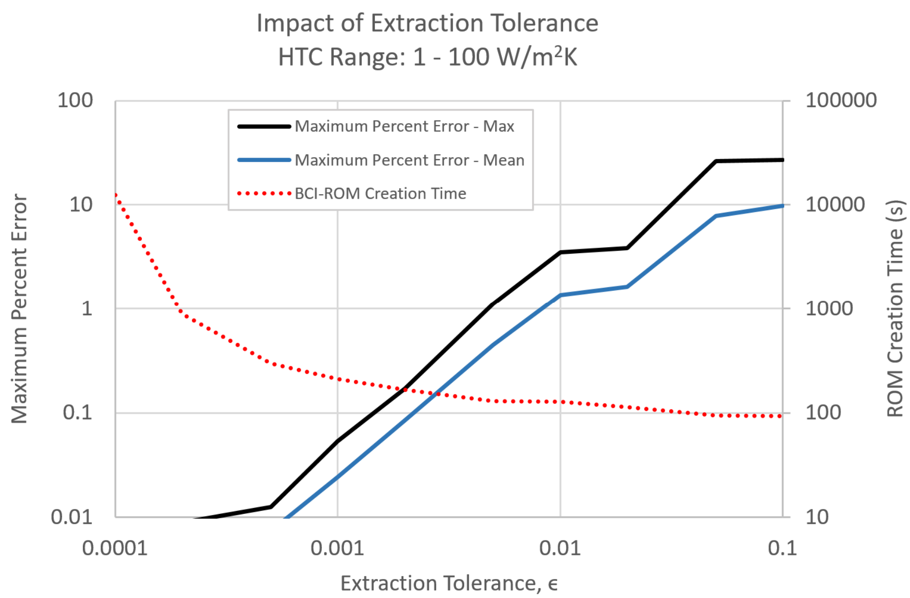


Figure 5 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100 W/m²K

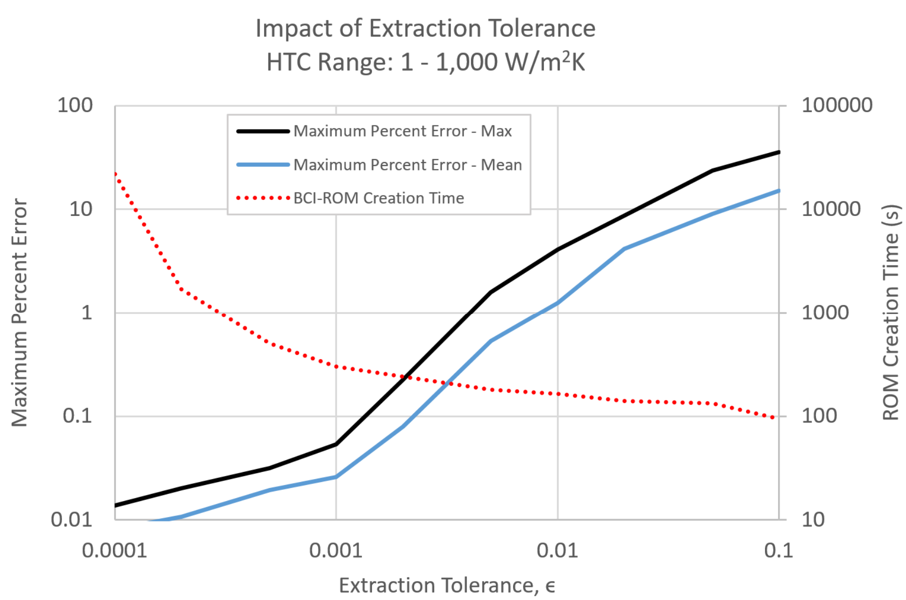


Figure 6 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1000 W/m²K

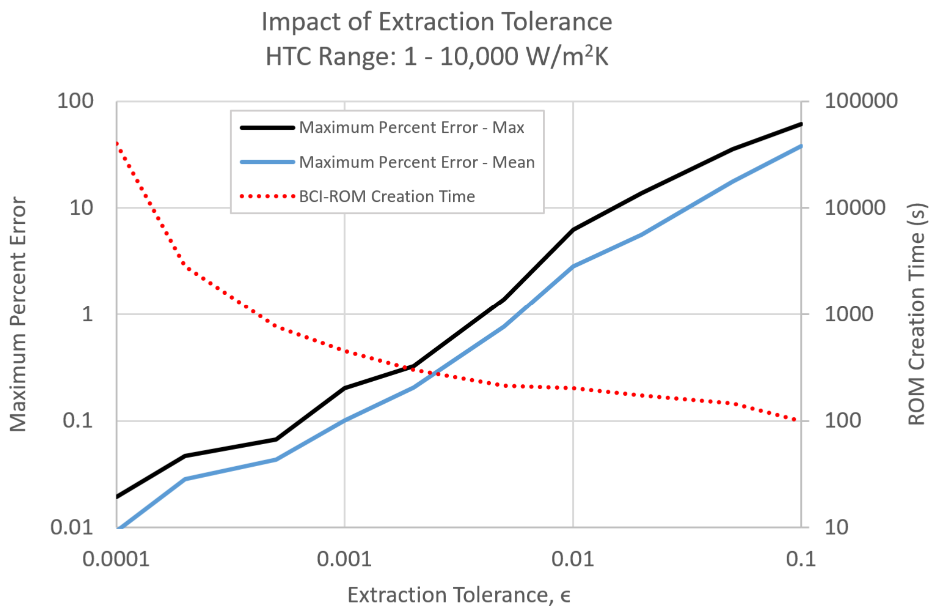


Figure 7 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-10,000 W/m²K

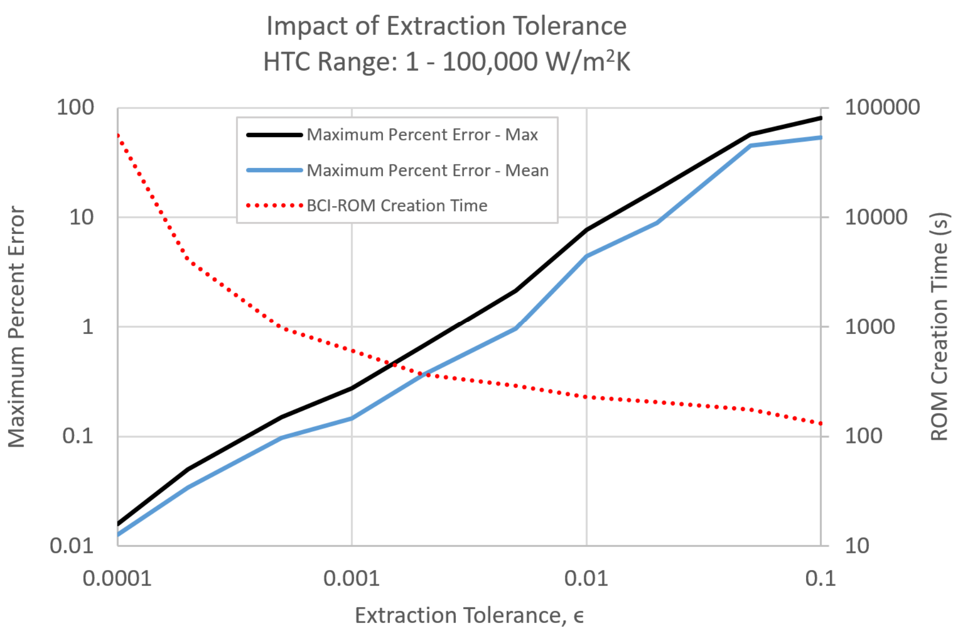


Figure 8 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100,000 W/m²K

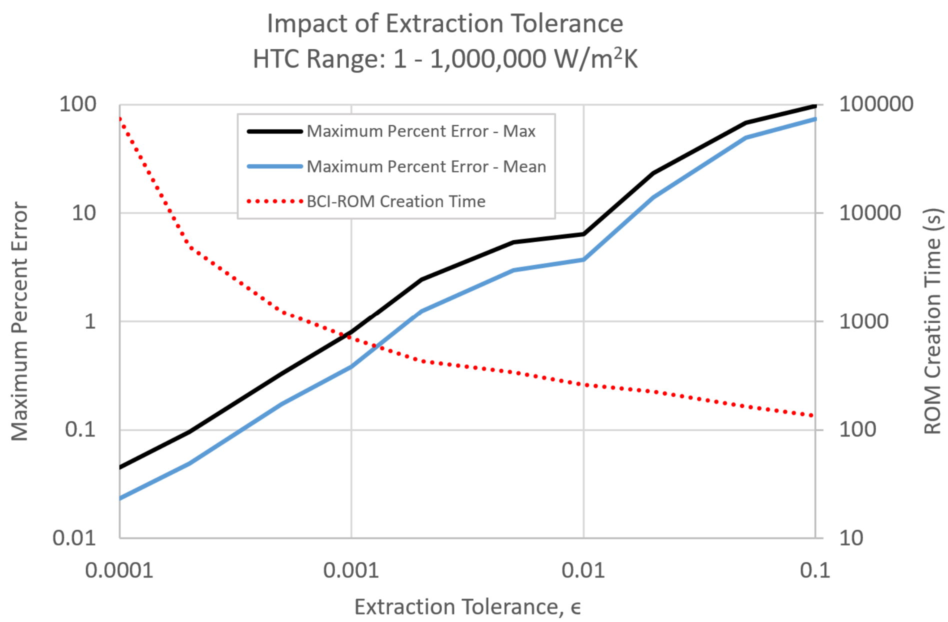


Figure 9 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1,000,000 W/m²K

Some observations from Figures 5-9:

- As the extraction tolerance ϵ decreases:
  - The maximum and mean errors decrease
  - The ROM creation time increases. The increase accelerates for $\epsilon < 0.001$
- As the extraction HTC range increases:
  - The ROM creation time increases, most notably at the smallest values of ϵ
  - There is little impact on maximum and mean errors

In Section 2 it is stated that the permitted range for the extraction tolerance is $0.0001 \leq \epsilon \leq 0.01$. This test case illustrates the rationale for imposing these bounds:

- Setting $\epsilon > 0.01$ fails to produce a usefully accurate BCI-ROM, with errors > 10%
- At $\epsilon = 0.0001$ the ROM creation calculation time is exponentially increasing, while gains in predictive accuracy are diminishing.

4.2 Detailed Flip Chip Ball Grid Array Package

4.2.1 Description of Model

The model contains a Flip Chip Ball Grid Array package explicitly including 576 solder balls. The die has two discrete zones of power dissipation included in one corner.

There are two ambient attributes defined to represent the heat flow path to a heatsink on the top of the package, and a PCB at the bottom. In the original model:

- The ambient attributes are defined as:
  - Heat Sink: HTC = 500 W/m^{2}K and $T_{ambient}$ = 0 degC. *[Attached to the top surface of the FC-BGA lid.]*
  - PCB: HTC = 50 W/m^{2}K and $T_{ambient}$ = 0 degC. *[Attached to the bottom faces of all the solder balls]*

  - Both heat sources are defined to dissipate 0.6 W.

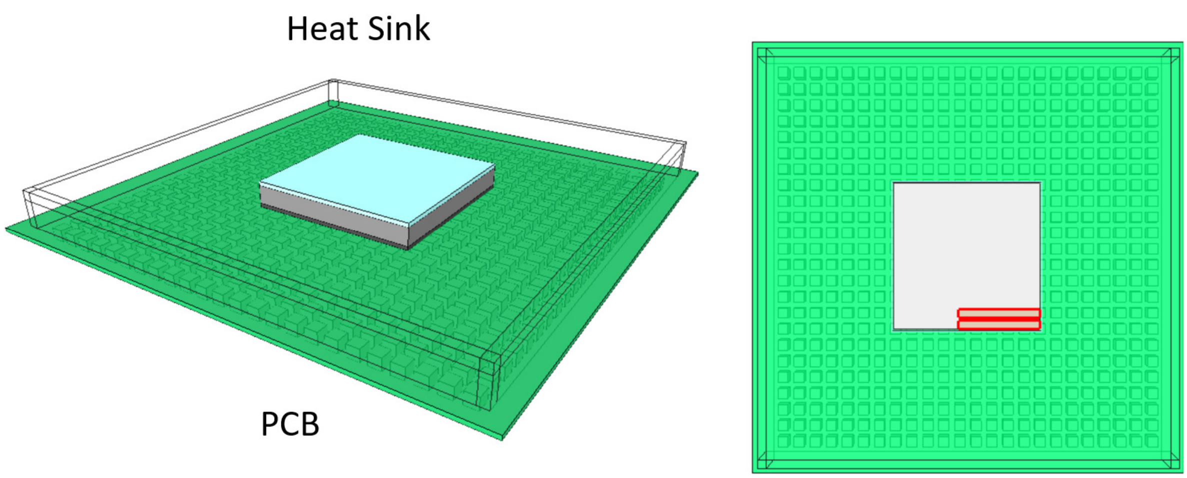


Figure 10: Flip Chip - BGA model. The labels approximately indicate the location of the attached ambients.

The model contains 792,180 grid cells. For validation purposes, both sources were turned on at t=0, and the transient duration set to 10,000s.

4.2.2 Model Variations

Command Center is used to create 30 scenarios with different combinations of HTC sets (using Design of Experiments). This was repeated for the following HTC ranges:

- HTC Range:
  - 1-100 W/m^{2}K
  - 1-1000 W/m^{2}K
  - 1-10,000 W/m^{2}K
  - 1-100,000 W/m^{2}K
  - 1-1,000,000 W/m^{2}K

4.2.3 Results: HTC Range: 1-10,000 and ϵ = 0.001

As described in Section 3, the full temperature response for each heat source monitor point is compared for every time step. The comparison for a single scenario is shown in Figure 11.

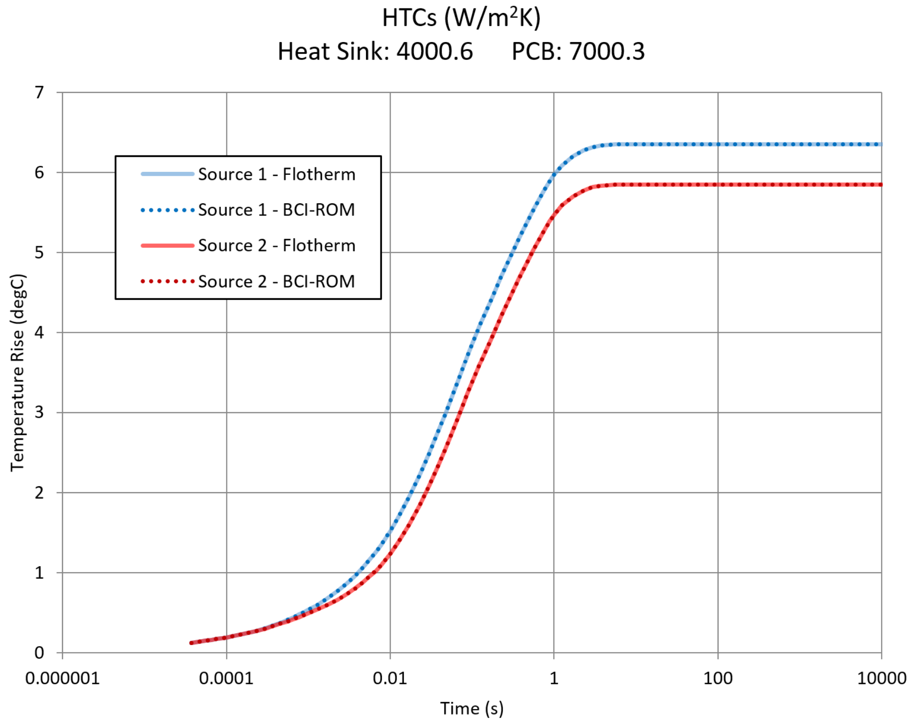


Figure 11– Maximum Percent Error = 0.0175%

Figure 12 reports the maximum percent error for each scenario tested. The maximum error encountered considering all scenarios was 0.445%, demonstrating the BCI nature of the ROM.

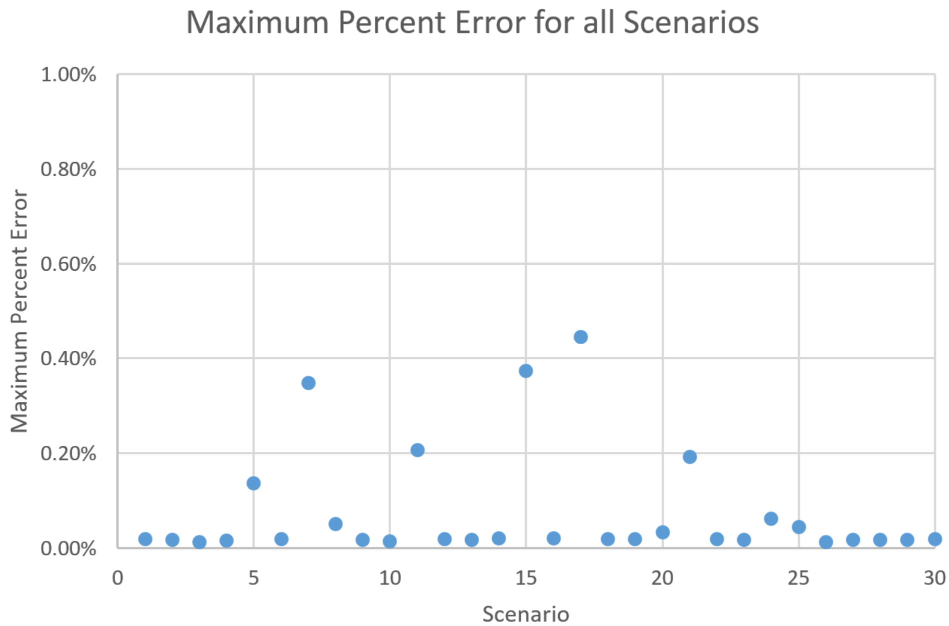


Figure 12 – Maximum Percent Errors for all Scenarios.
HTC Range = 1 – 10,000 W/m²K and ϵ = 0.001
Max = 0.445%, Mean = 0.0749%, Standard Deviation = 0.437%

4.2.4 Impact of Extraction Tolerance, ϵ

The impact of the extraction tolerance, ϵ, on BCI-ROM accuracy and creation time for this model for various HTC ranges is shown in Figures 13-17.

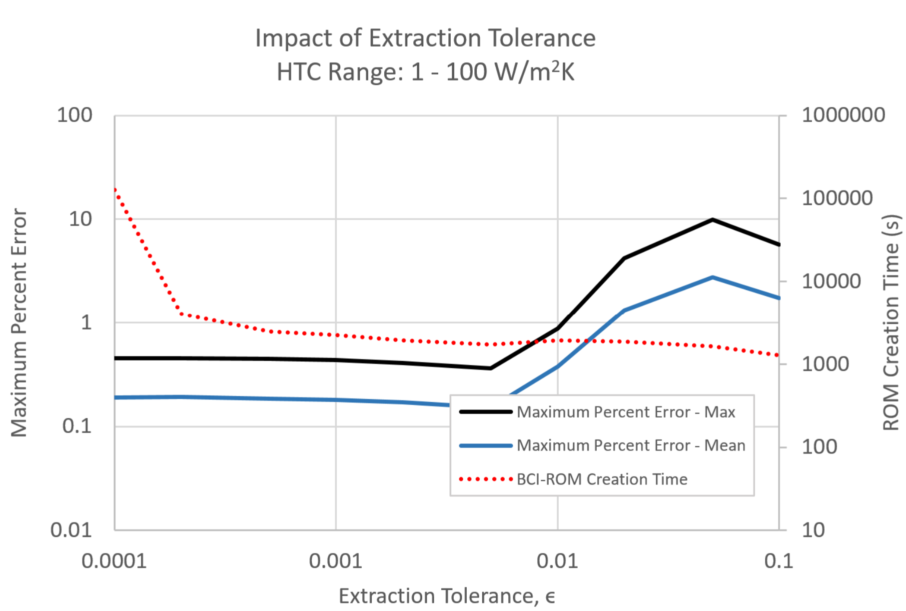


Figure 13 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100 W/m²K

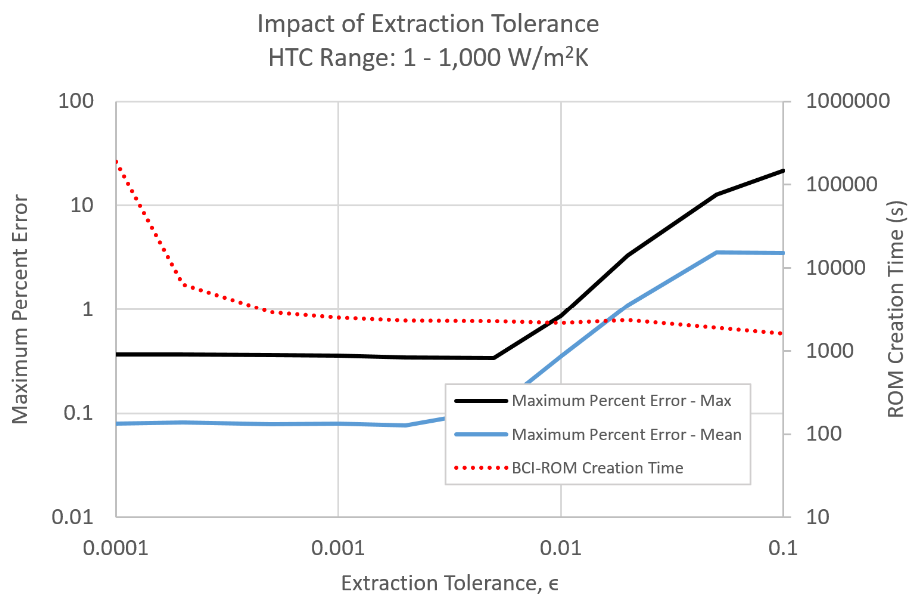


Figure 14 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1000 W/m²K

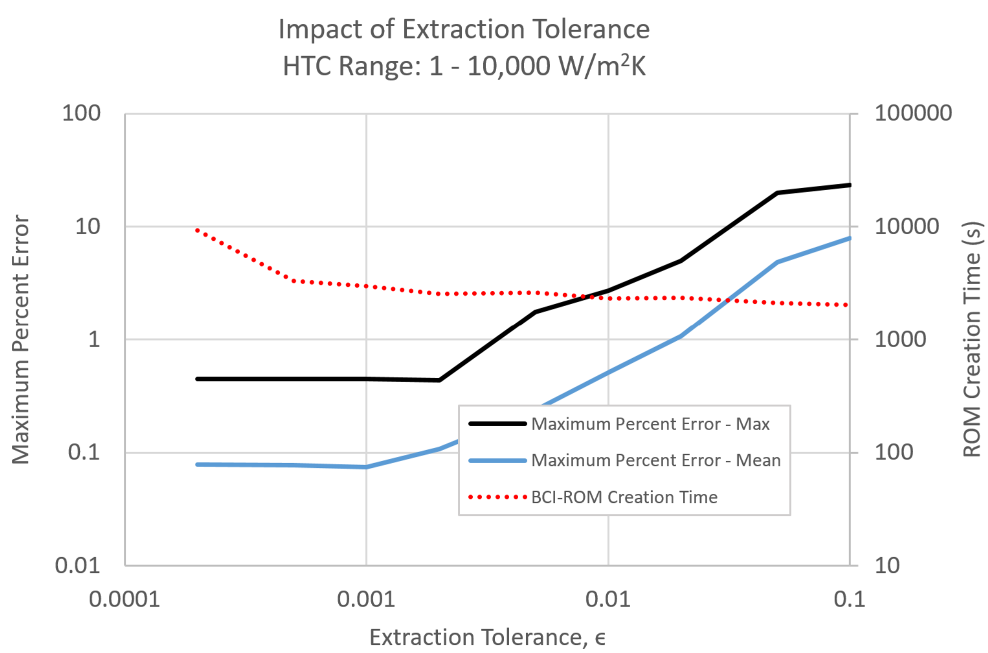


Figure 15 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-10,000 W/m²K

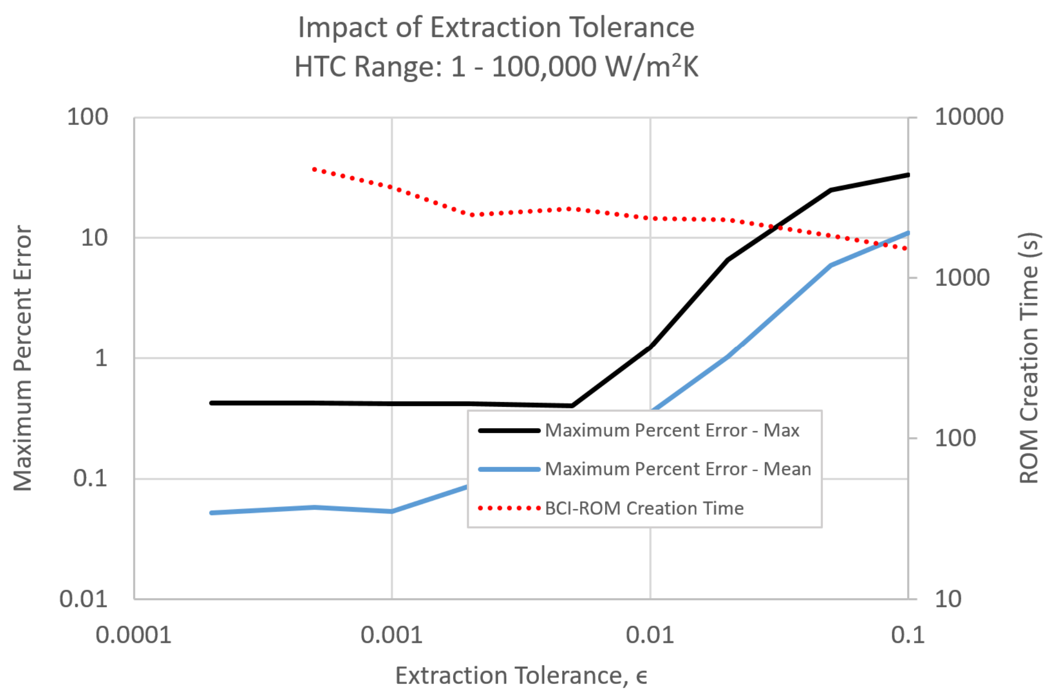


Figure 16 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100,000 W/m²K

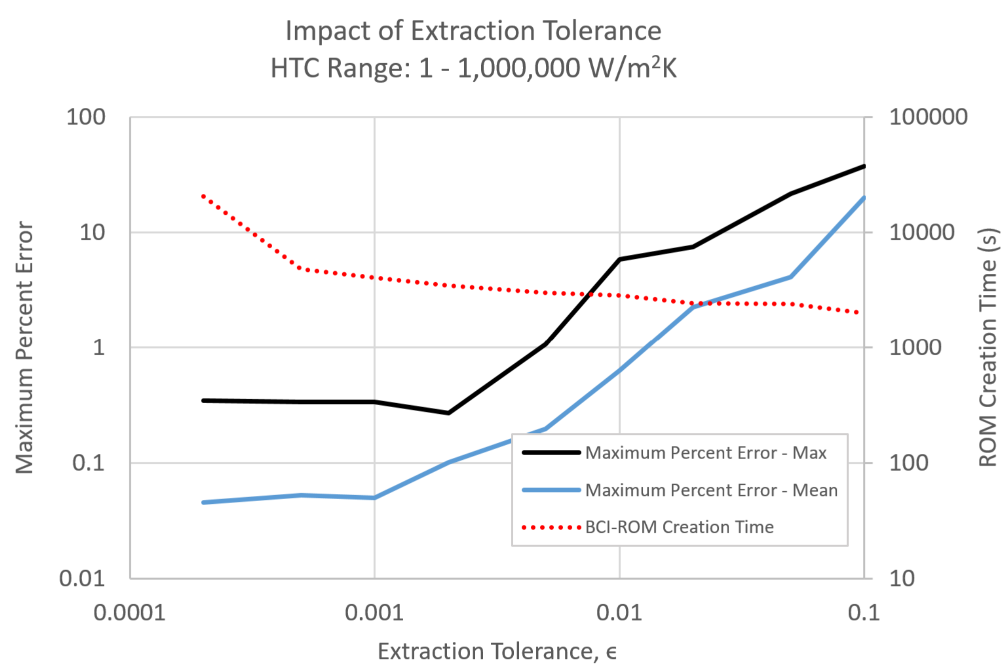


Figure 17 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1,000,000 W/m²K

Some observations from Figures 13-17:

- As the extraction tolerance ϵ decreases:
  - The maximum and mean errors decreases until roughly 0.001.
  - The ROM creation time increases.
- As the extraction HTC range increases:

- The ROM creation time increases, most notably at the smallest values of ϵ
- There is little impact on maximum and mean errors

In Section 2 it is stated that the permitted range for the extraction tolerance is $0.0001 \leq \epsilon \leq 0.01$. This test case illustrates the rationale for imposing these bounds:

- Setting $\epsilon > 0.01$ fails to produce a usefully accurate BCI-ROM, with errors > 10%
- At $\epsilon = 0.0001$ the ROM creation calculation time is exponentially increasing, while gains in predictive accuracy are diminishing. For the larger HTC ranges, the ROM creation process failed to complete in a practical amount of time. Those points are excluded from the figures.

4.3 IGBT Module

4.3.1 Description of Model

The model contains a IGBT module with 18 heat sources representing 9 IGBT die and diode pairs.

There are three ambient attributes defined to represent the heat flow path to a cold plate at the bottom of the module, and natural convection on the top and sides of the module. In the original model:

- The ambient attributes are defined as:
  - <u>Cold Plate:</u> HTC = 1000 W/m^{2}K and $T_{ambient}$ = 0 degC. *[Attached to the bottom surface of the module.]*
  - <u>Top:</u> HTC = 11 W/m^{2}K and $T_{ambient}$ = 0 degC. *[Attached to the top face of the module]*
  - <u>Outside World:</u> HTC = 10 W/m^{2}K and $T_{ambient}$ = 0 degC. *[Attached to all other external faces of the module]*

  - Cumulative power dissipation for the 18 heat sources in the original model is 111 W.

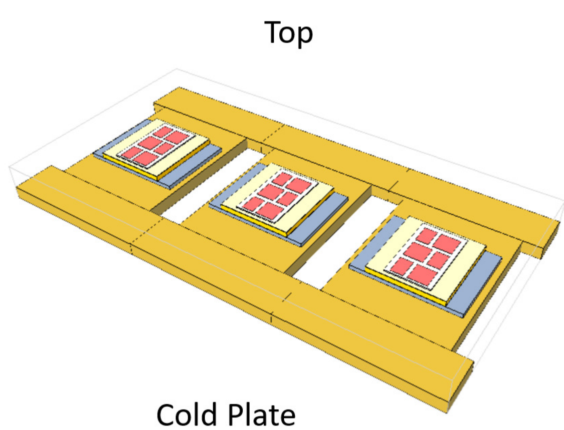


Figure 18: IGBT module model. The labels approximately indicate the location of the attached ambients.

The model contains 580,032 grid cells. For validation purposes, both sources were turned on at t=0, and the transient duration set to 200s.

4.3.2 Model Variations

Command Center is used to create 60 scenarios with different combinations of HTC sets (using Design of Experiments). This was repeated for the following HTC ranges:

- HTC Range:
  - 1-100 W/m²K
  - 1-1000 W/m²K
  - 1-10,000 W/m²K
  - 1-100,000 W/m²K
  - 1-1,000,000 W/m²K

4.3.3 Results: HTC Range: 1-10,000 and ϵ = 0.001

As described in Section 3, the full temperature response for each heat source monitor point is compared for every time step. The comparison for a single scenario is shown in Figures 19-20. The data is split into two charts for improved clarity.

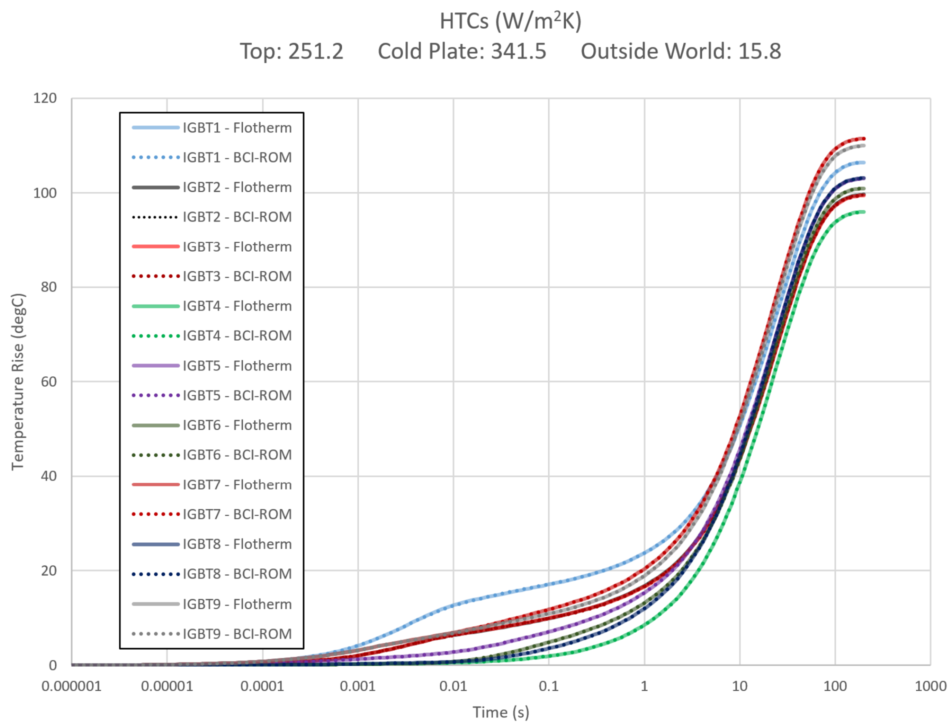


Figure 19– IGBT Die Temperature Responses: Maximum Percent Error = 0.151%

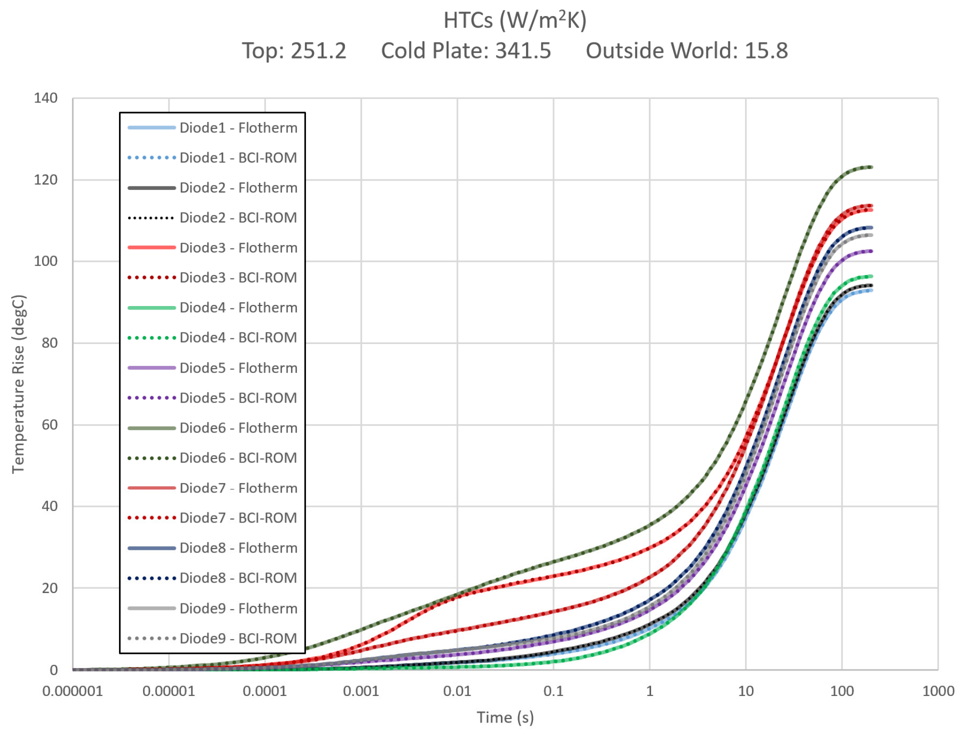


Figure 20– Diode Temperature Responses: Maximum Percent Error = 0.151%

Figure 21 reports the maximum percent error for each scenario tested. The maximum error encountered considering all scenarios was 0.558%, demonstrating the BCI nature of the ROM.

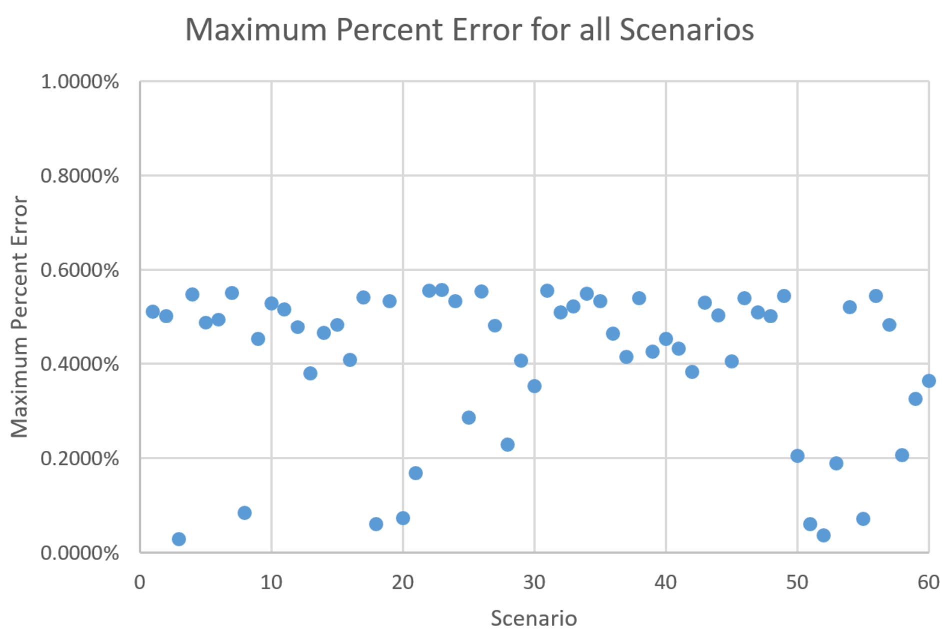


Figure 21 – Maximum Percent Errors for all Scenarios.
HTC Range = 1 – 10,000 W/m²K and ϵ = 0.001
Max = 0.558%, Mean = 0.409%, Standard Deviation = 0.00161%

4.3.4 Impact of Extraction Tolerance, ϵ

The impact of the extraction tolerance, ϵ, on BCI-ROM accuracy and creation time for this model for various HTC ranges is shown in Figures 22-26.

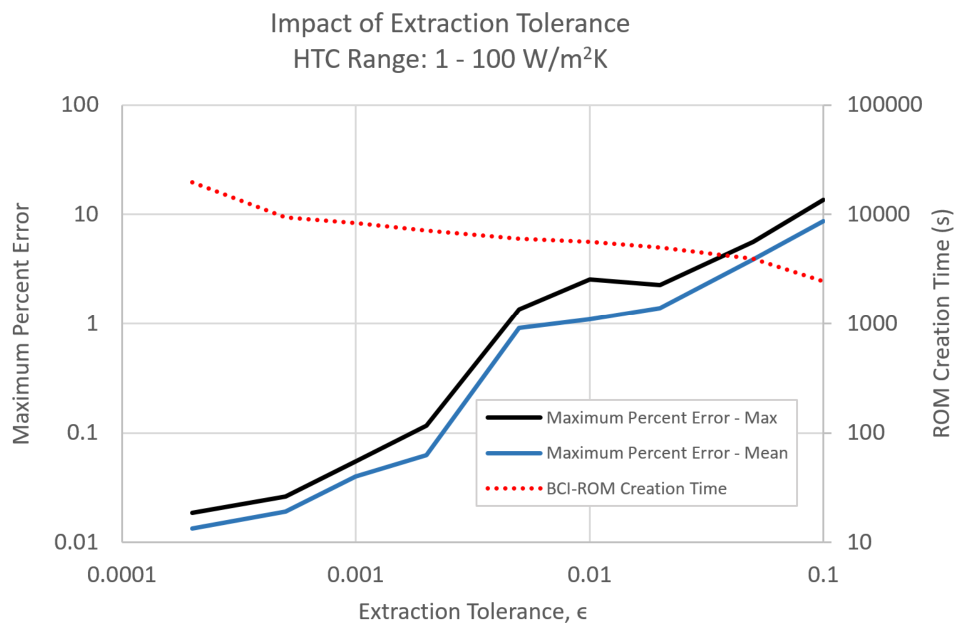


Figure 22 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100 W/m^2K

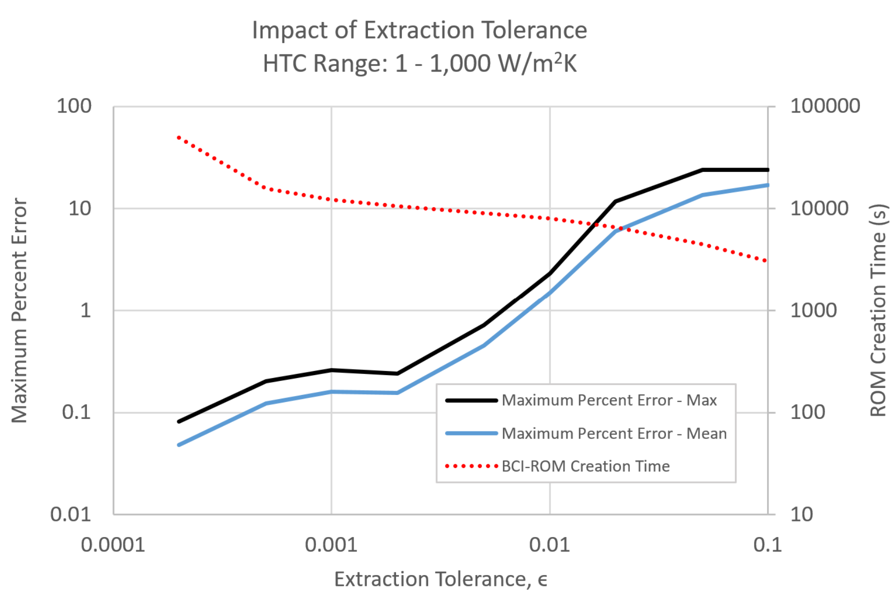


Figure 23 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1000 W/m^2K

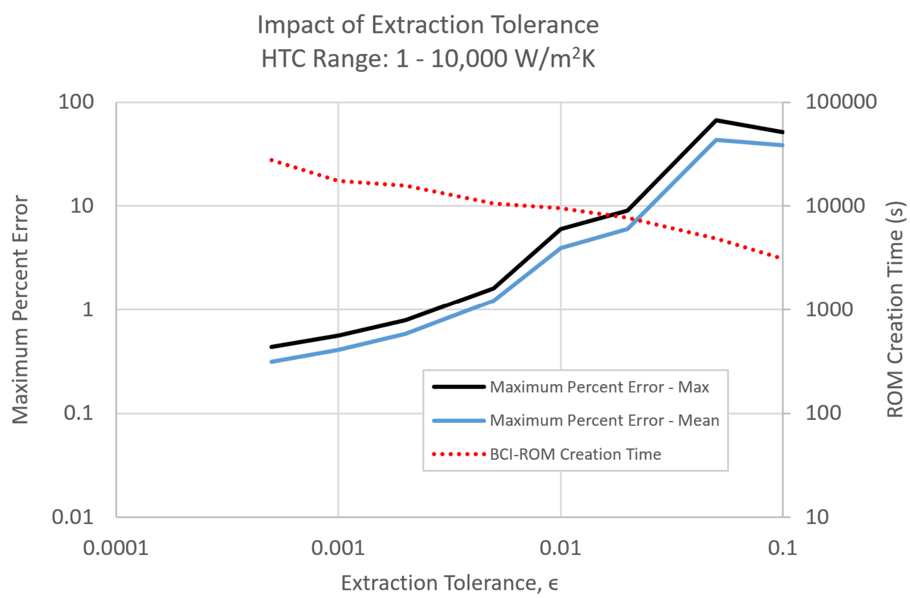


Figure 24 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-10,000 W/m²K

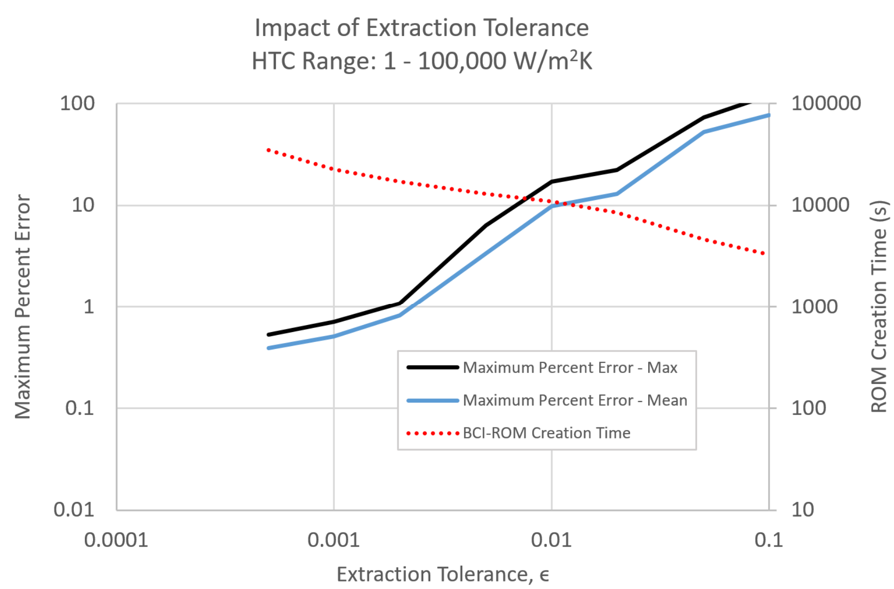


Figure 25 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100,000 W/m²K

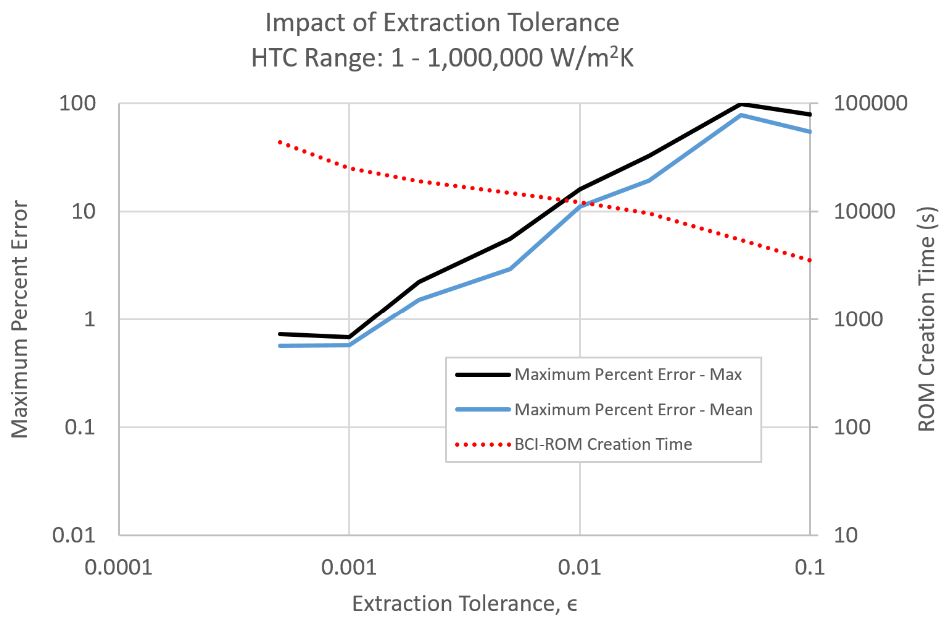


Figure 26 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1,000,000 W/m²K

Some observations from Figures 22-26:

- As the extraction tolerance ϵ decreases:
    - The maximum and mean errors decreases
    - The ROM creation time increases.
- As the extraction HTC range increases:
    - The ROM creation time increases, most notably at the smallest values of ϵ
    - There is little impact on maximum and mean errors

In Section 2 it is stated that the permitted range for the extraction tolerance is $0.0001 \leq \epsilon \leq 0.01$. This test case illustrates the rationale for imposing these bounds:

- Setting ϵ > 0.01 fails to produce a usefully accurate BCI-ROM, with errors > 10%
- For the larger HTC ranges, the ROM creation process failed to complete for ϵ < 0.002 in a practical amount of time. Those points are excluded from the figures.

4.4 Multi-Die SOIC on Test PCB

4.4.1 Description of Model

The model contains a SOIC package mounted on a 4-layer test PCB. The die has two discrete zones of power dissipation. The traces on the top layer of the PCB are modeled explicitly.

There are three ambient attributes defined and attached to a large number of cutouts that surround the entire model. In the original model:

- The ambient attributes are defined as:
  - Z High: HTC = 14 W/m²K and $T_{ambient}$ = 0 degC. *[Attached to all surfaces of PCB and component on the top side of the board]*
  - Sides: HTC = 12 W/m²K and $T_{ambient}$ = 0 degC. *[Attached to all edges of the PCB]*
  - Z Low: HTC = 8 W/m²K and $T_{ambient}$ = 0 degC. *[Attached to the bottom face of the PCB]*

  - Both heat sources are defined to dissipate 0.65 W.

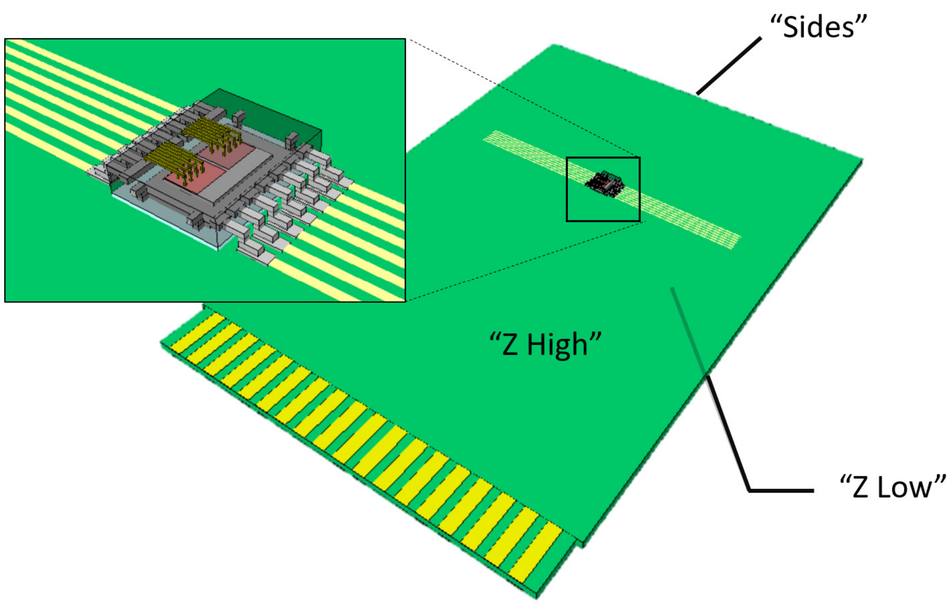


Figure 27: SOIC on a test PCB. The labels approximately indicate the location of the attached ambients.

The model contains 914,204 grid cells. For validation purposes, both sources were turned on at t=0, and the transient duration set to 10,000s.

4.4.2 Model Variations

Command Center is used to create 40 scenarios with different combinations of HTC sets (using Design of Experiments). This was repeated for the following HTC ranges:

  - HTC Range:
    - 1-100 W/m²K
    - 1-1000 W/m²K

- 1-10,000 W/m²K
- 1-100,000 W/m²K
- 1-1,000,000 W/m²K

4.4.3 Results: HTC Range: 1-10,000 and ϵ = 0.001

As described in Section 3, the full temperature response for each heat source monitor point is compared for every time step. The comparison for a single scenario is shown in Figure 28.

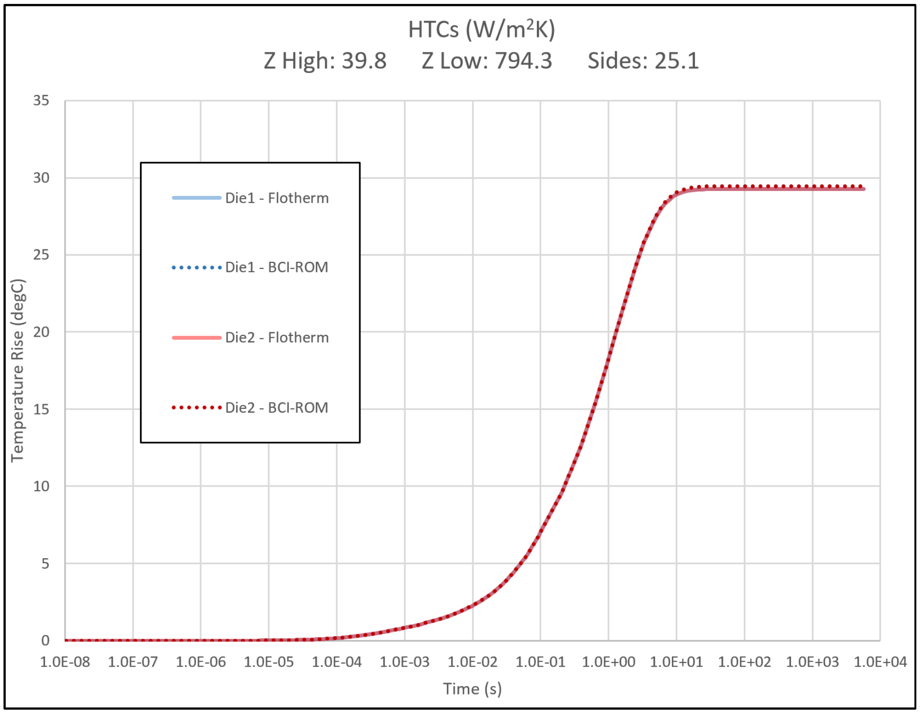


Figure 28 – Maximum Percent Error = 0.618%

Figure 29 reports the maximum percent error for each scenario tested. The maximum error encountered considering all scenarios was 0.953%, demonstrating the BCI nature of the ROM.

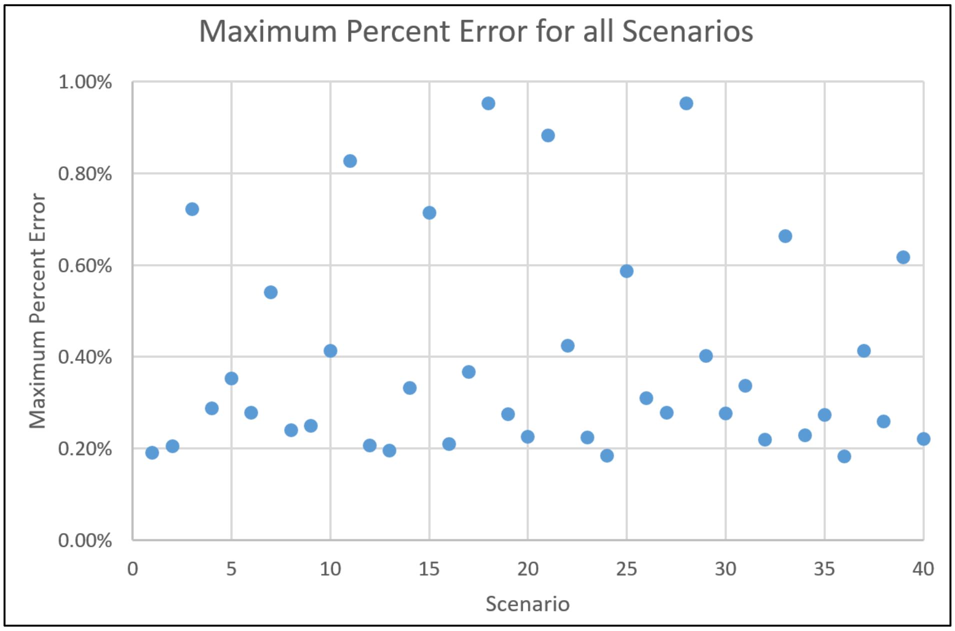


Figure 29 – Maximum Percent Errors for all Scenarios.
HTC Range = 1 – 10,000 W/m²K and ϵ = 0.001
Max = 0.953%, Mean = 0.393%, Standard Deviation = 0.169%

4.4.4 Impact of Extraction Tolerance, ϵ

The impact of the extraction tolerance, ϵ, on BCI-ROM accuracy and creation time for this model for various HTC ranges is shown in Figures 30-34.

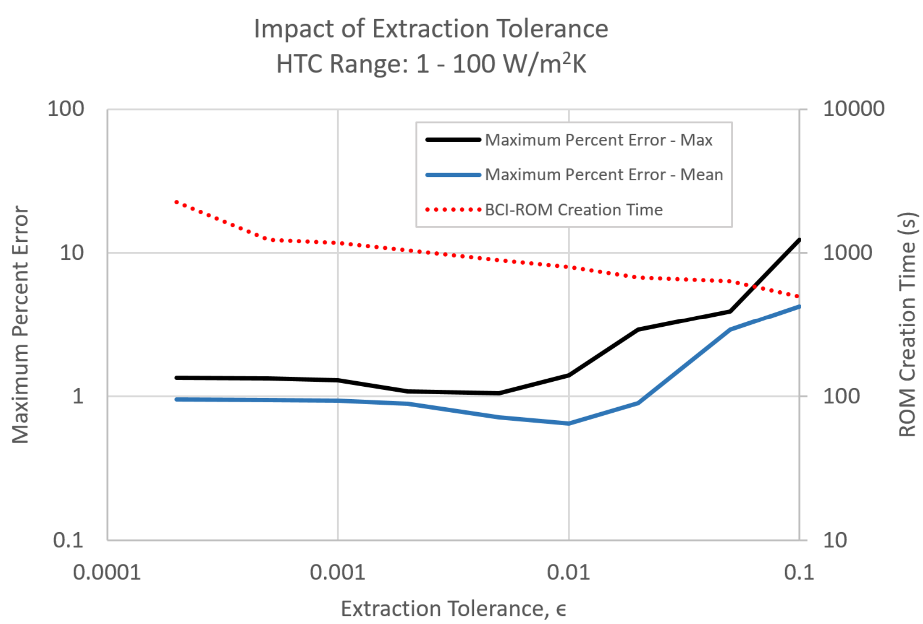


Figure 30 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100 W/m²K

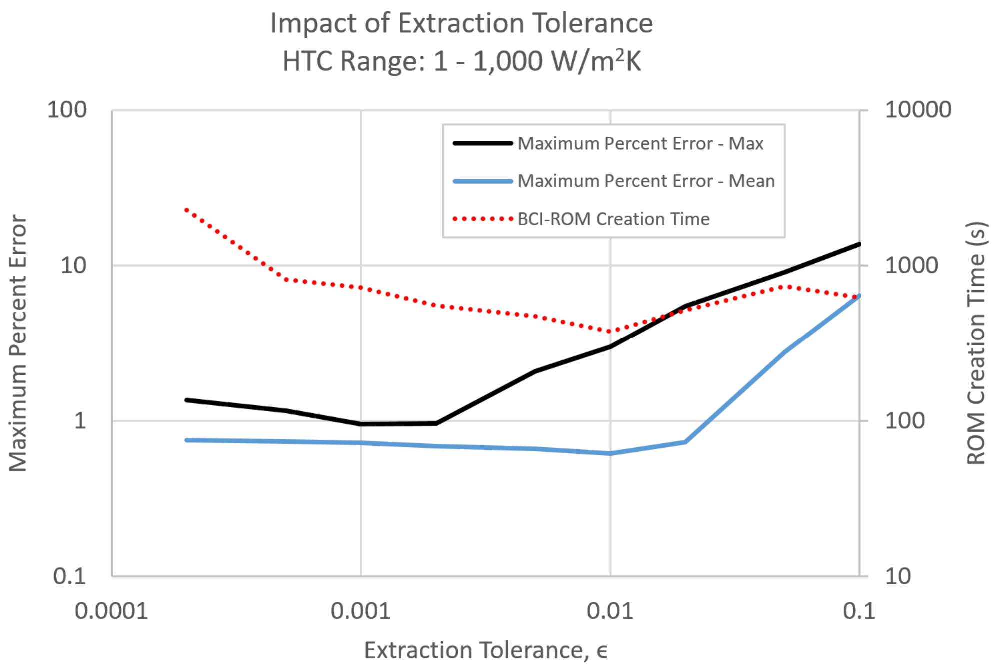


Figure 31 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1000 W/m²K

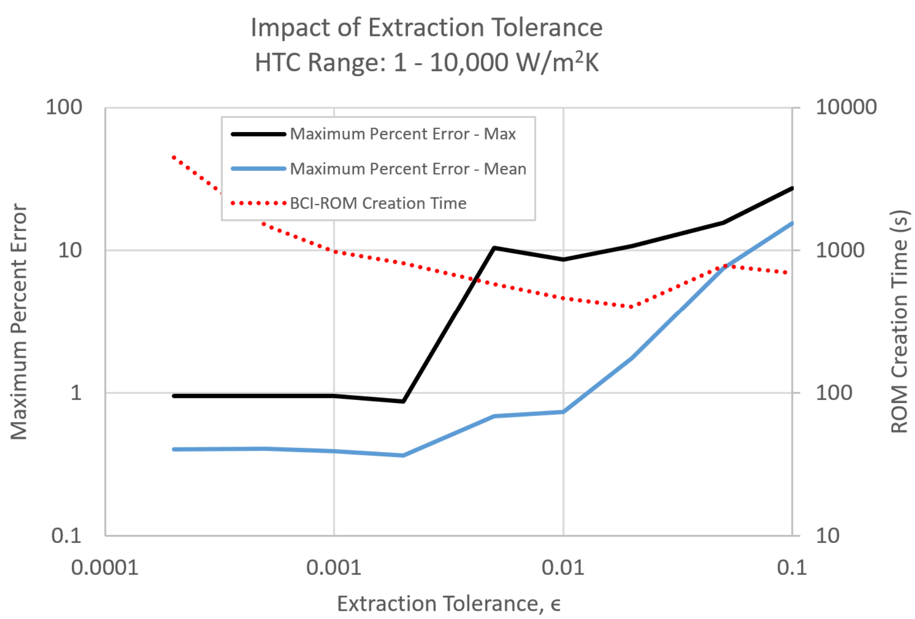


Figure 32 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-10,000 W/m²K

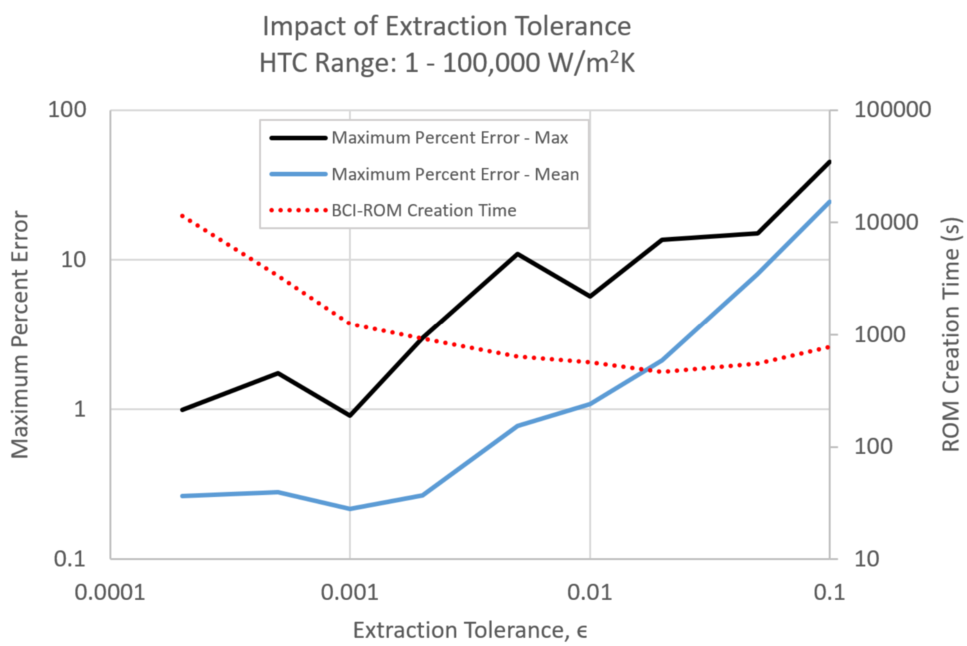


Figure 33 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100,000 W/m²K

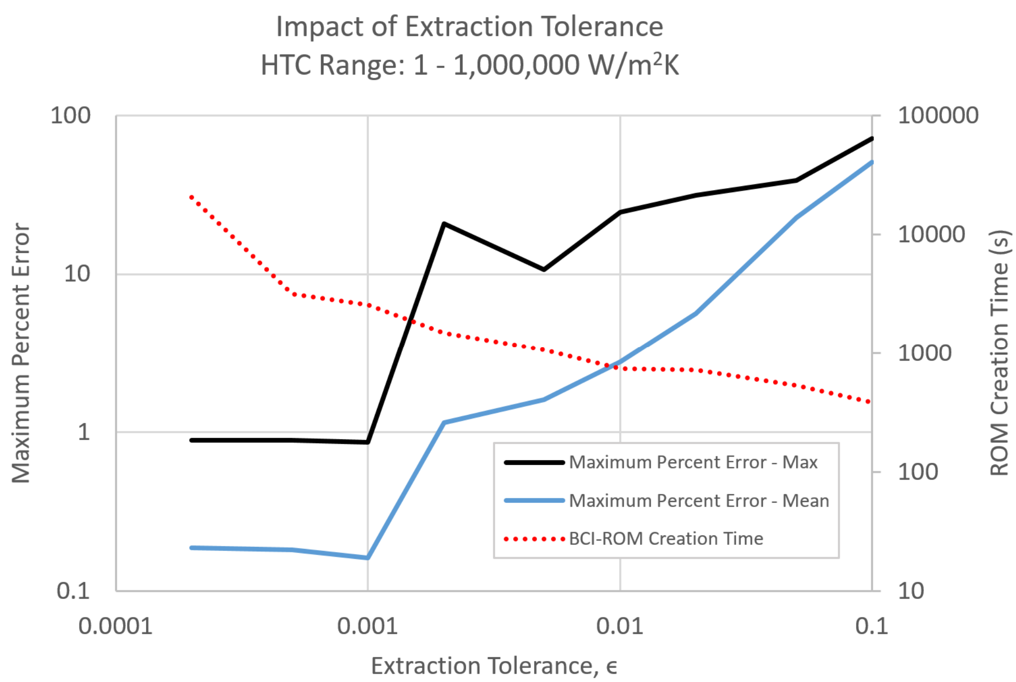


Figure 34 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1,000,000 W/m²K

Some observations from Figures 30-34:

- As the extraction tolerance ϵ decreases:
  - The maximum and mean errors decreases, though not monotonically in this case.
  - The ROM creation time increases.
- As the extraction HTC range increases:
  - The ROM creation time increases, most notably at the smallest values of ϵ
  - The largest HTC ranges required ϵ < 0.002 to achieve errors below 1%.

In Section 2 it is stated that the permitted range for the extraction tolerance is $0.0001 \leq \epsilon \leq 0.01$. This test case illustrates the rationale for imposing these bounds:

- Setting $\epsilon > 0.01$ fails to produce a usefully accurate BCI-ROM, with errors > 10%
- At $\epsilon = 0.0002$ the ROM creation calculation time is exponentially increasing, while gains in predictive accuracy are diminishing.

4.5 QFN Package in JEDEC Ring Cold Plate Apparatus

4.5.1 Description of Model

The model contains a QFN package mounted in a JEDEC Ring Cold Plate test apparatus. The die is modeled with a single heat source. The traces on the top layer of the PCB are modeled explicitly.

There is one ambient attribute defined and attached to a series of cutout objects that represent the cold plate coolant loop. In the original model:

- The ambient attribute is defined as:
    - Coolant: HTC = 100000 W/m^{2}K and $T_{ambient}$ = 0 degC. *[Attached to all surfaces of the cutout objects]*

    - The heat sources is defined to dissipate 0.75 W.

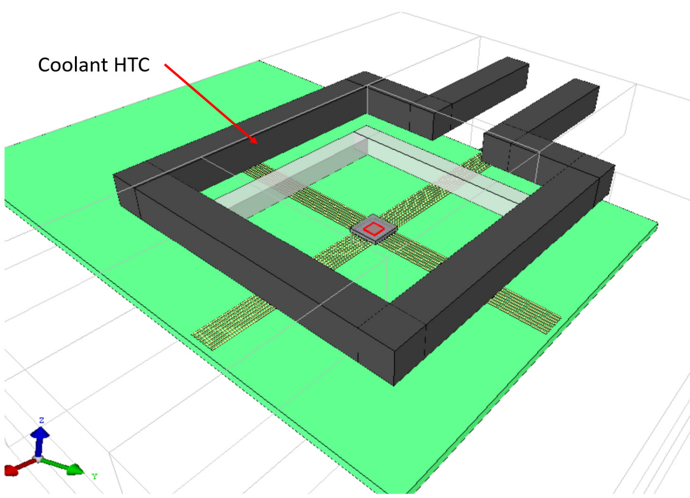


Figure 35: Model of QFN package on a PCB within a cold plate apparatus.

The model contains 263,744 grid cells. For validation purposes, the source was turned on at t=0, and the transient duration set to 100,000s.

4.5.2 Model Variations

Command Center is used to create 40 scenarios with different combinations of HTC sets (using Design of Experiments). This was repeated for the following HTC ranges:

- HTC Range:
    - 1-100 W/m²K
    - 1-1000 W/m²K
    - 1-10,000 W/m²K
    - 1-100,000 W/m²K
    - 1-1,000,000 W/m²K

4.5.3 Results: HTC Range: 1-10,000 and ϵ = 0.001

As described in Section 3, the full temperature response for each heat source monitor point is compared for every time step. The comparison for a single scenario is shown in Figure 36.

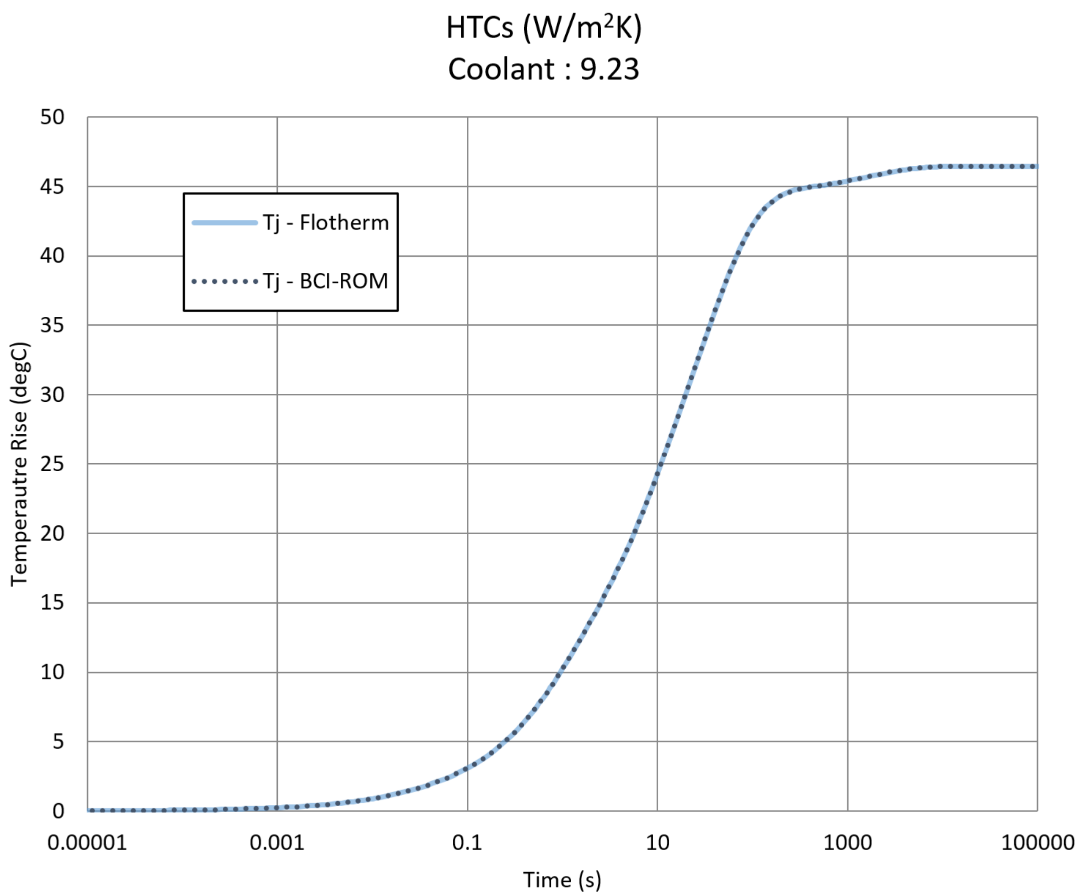


Figure 36– Maximum Percent Error = 0.0958%

Figure 37 reports the maximum percent error for each scenario tested. The maximum error encountered considering all scenarios was 0.325%, demonstrating the BCI nature of the ROM.

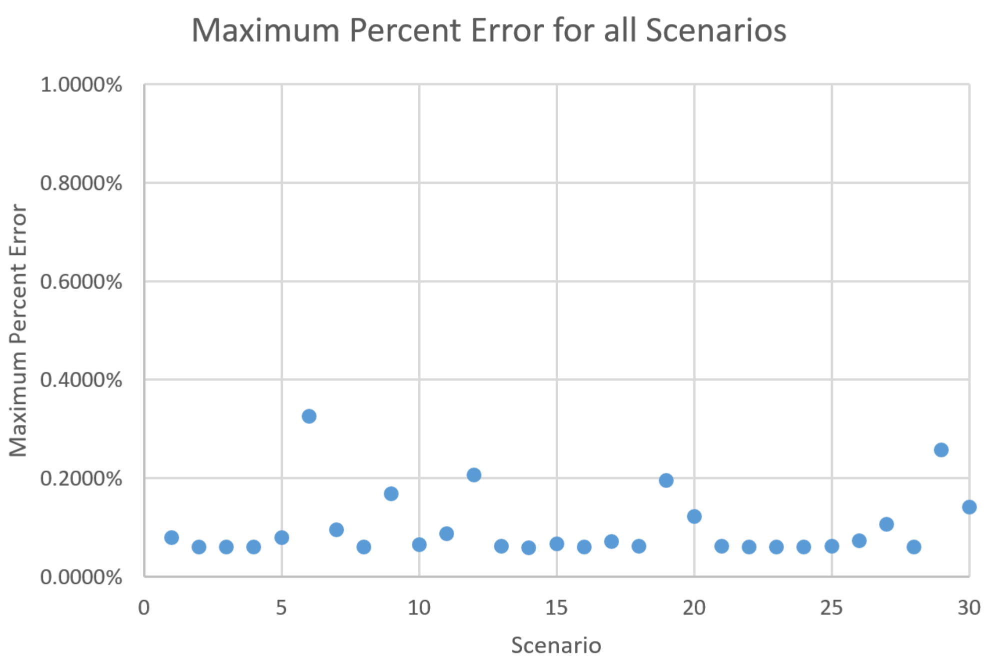


Figure 37 – Maximum Percent Errors for all Scenarios.
HTC Range = 1 – 10,000 W/m^{2}K and ϵ = 0.001
Max = 0.325%, Mean = 0.0996%, Standard Deviation = 0.0305%

4.5.4 Impact of Extraction Tolerance, ϵ

The impact of the extraction tolerance, ϵ, on BCI-ROM accuracy and creation time for this model for various HTC ranges is shown in Figures 38-42.

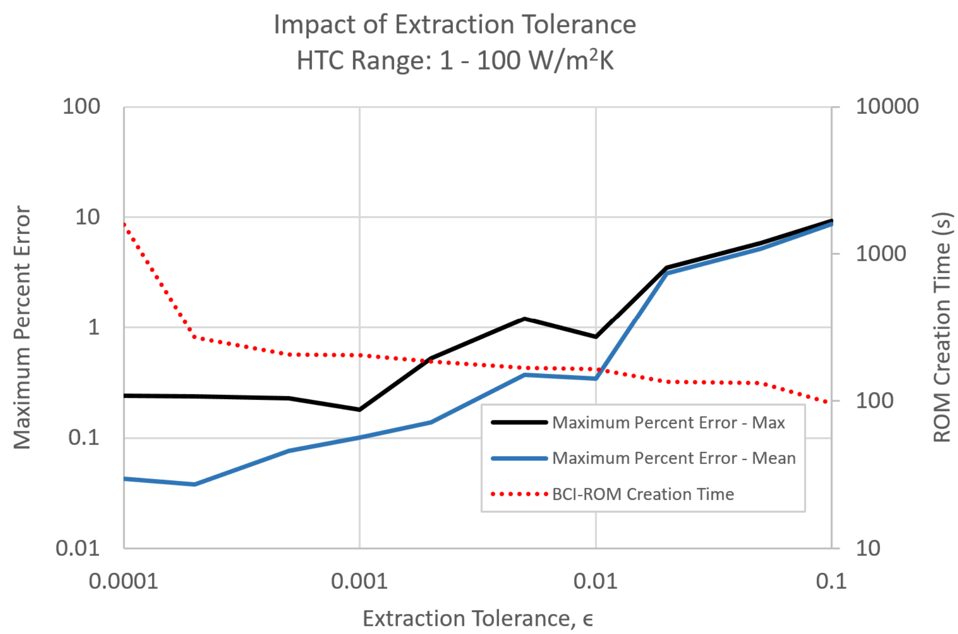


Figure 38 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100 W/m²K

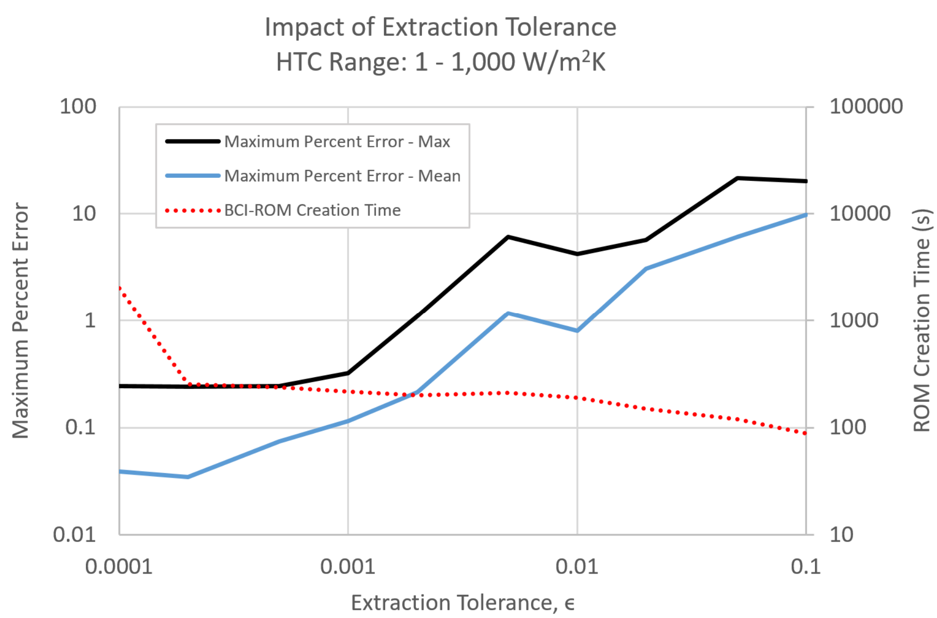


Figure 39 - Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1000 W/m²K

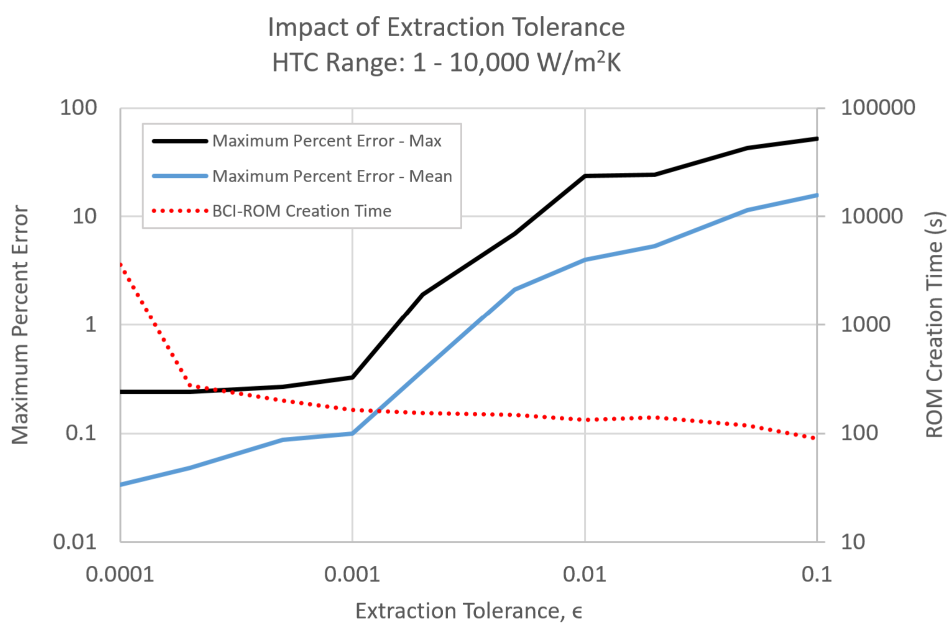


Figure 40 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-10,000 W/m²K

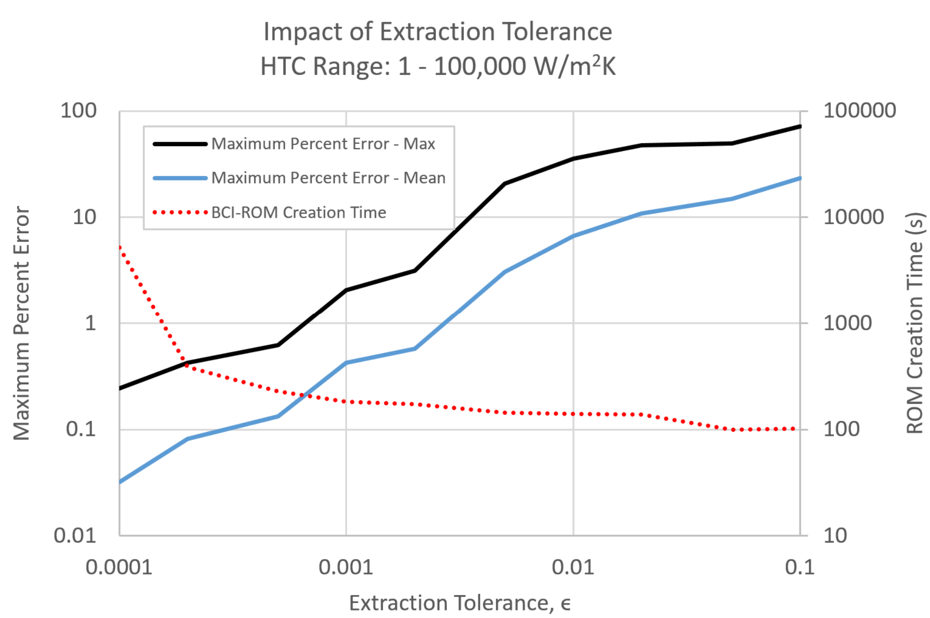


Figure 41 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-100,000 W/m²K

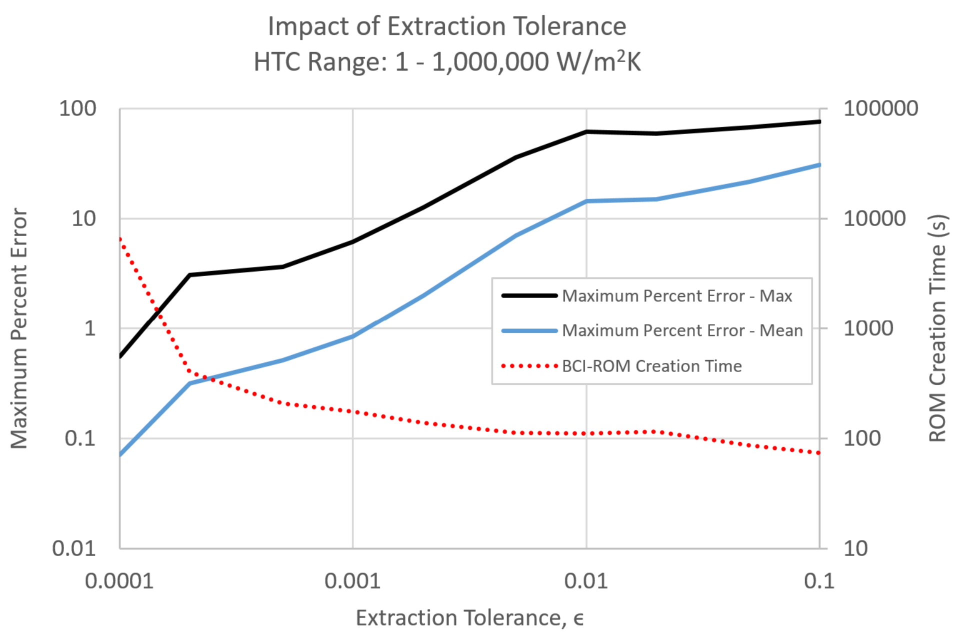


Figure 42 – Errors and BCI-ROM creation time vs ϵ, for HTC Range 1-1,000,000 W/m²K

Some observations from Figures 38-42:

- As the extraction tolerance ϵ decreases:
  - The maximum and mean errors decreases
  - The ROM creation time increases.
- As the extraction HTC range increases:
  - The ROM creation time increases, most notably at the smallest values of ϵ
  - There is little impact on maximum and mean errors

In Section 2 it is stated that the permitted range for the extraction tolerance is $0.0001 \leq \epsilon \leq 0.01$. This test case illustrates the rationale for imposing these bounds:

- Setting $\epsilon > 0.01$ fails to produce a usefully accurate BCI-ROM, with errors > 10%
- At the largest HTC range, using $\epsilon = 0.0001$ was necessary to reduce the maximum error below 1%.

5.0 Conclusions

The FANTASTIC method for Simcenter Flotherm was evaluated for five models, ranging in scale from package to module to package in system. The ROMs generated were demonstrated to be BCI and produce consistently high levels of accuracy. For the default extraction values of $\epsilon = 0.001$ and HTC range of 1-10,000 W/m²K the average percent error observation across 200 scenarios and 25 heat sources (5000 Temperature vs time curves) was 0.248%, with a maximum under 1%. These numbers are aligned well with the results reported in the literature[1,2,3,4].

6.0 References

1. L. Codecasa, D. D'Amore, and P. Maffezzoni, “Parameters for multi-point moment matching reduction of discretized thermal networks”, *in Proc. IEEE THERMINIC, 2002, pp. 151-154*.
2. L Codecasa, V d'Alessandro, A Magnani, N Rinaldi, PJ Zampardi, “Fast novel thermal analysis simulation tool for integrated circuits (FANTASTIC)”, *20th International Workshop on Thermal Investigations of ICs and Systems (THERMINIC) article 6972507 United Kingdom, 2014*.
3. Lorenzo Codecasa, Vincenzo d'Alessandro, Alessandro Magnani, Niccolò Rinaldi, “Matrix reduction tool for creating boundary condition independent dynamic compact thermal models”, *Thermal Investigations of ICs and Systems (THERMINIC) 2015 21st International Workshop on, pp. 1-5, 2015*.
4. JHJ Janssen, L Codecasa, “Why matrix reduction is better than objective function based optimization in compact thermal model creation”, *Thermal Investigations of ICs and Systems (THERMINIC) 2015 21st International Workshop on, pp. 1-6, 2015*.