散熱片的暫態分析理論推導與數值分析

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CONTENT

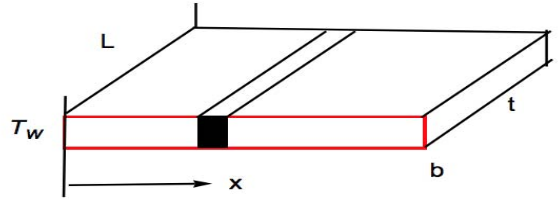
- Research motivation– transient solver correct??
- Energy equation PDE derivation
- Mathematically solving 1D T(x,t) by separation variable
- Numerically solving 1D T(x,t) by Flotherm
- Conclusion

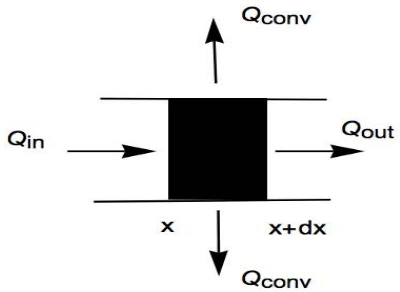
RESEARCH MOTIVATION

- Changes of electronic cooling eco system
  - iPad/iPhone corrodes PC market share

- Cloud server function evolution towards energy saving -> idle constantly
- Power-up suddenly for complex tasks
  - Heat sink transient phenomenon needs to study for reliability

SCHEMATICS OF FIN PROFILE EQUATIONS
STEADY STATE SOLUTION





$$Q_{in} - Q_{out} + Q_g = Q_{st} \implies Q_{in} - Q_{out} = 0$$

$$\implies Q_{cond-net} + Q_{conv-net} = 0$$

$$Q_{cond-net} = Q_{in} - Q_{out} = Q_x - Q_{x+dx}$$

$$\implies -kA \frac{\partial^2 T}{\partial x^2} dx = h\,p\,dx\,(T - T_\infty)$$

$$Q_{cond-net} + Q_{conv-net} = 0$$

$$\implies -kA \frac{\partial^2 T}{\partial x^2} dx + h\,p\,dx\,(T - T_\infty) = 0$$

$$kA \frac{\partial^2 T}{\partial x^2} - h\,p\,(T - T_\infty) = 0$$

$$\frac{\partial^2 T}{\partial x^2} - \frac{hp}{kA}(T - T_\infty) = 0$$

BCs

$$(1)\ T(0) = T_b \qquad (2)\ \left.\frac{\partial T}{\partial X}\right|_{x=1} = 0$$

To nondimensionlize the following equation:

$$\text{Let } \theta = \frac{T - T_a}{T_w - T_a} \quad X = \frac{x}{b} \quad \frac{hp}{kA} = m^2 \quad M = \sqrt{\frac{hp}{kA}}\, b$$

$$\frac{\partial^2 T}{\partial x^2} - \frac{hp}{kA}(T - T_\infty) = 0$$

$$\frac{(T_w - T_\infty)}{b^2} \frac{\partial^2 \theta}{\partial X^2} - m^2\,((T_w - T_\infty)\,\theta + T_\infty - T_\infty) = 0$$

$$\implies \frac{\partial^2 \theta}{\partial X^2} - M^2\,\theta = 0$$

BCs

$$(1)\ \theta(0) = 1, \quad (2)\ \left.\frac{\partial \theta}{\partial X}\right|_{x=1} = 0$$

$$\theta(X) \to \frac{\text{Cosh}\,(M\,(1-X))}{\text{Cosh}\,(M)}$$

FLOTHERM STEADY STATE RESULT VALIDATED BY THEORETICAL RESULT

Dimension : $t = 1\text{mm}$ $L = 108\text{mm}$ $b = 40\text{mm}$

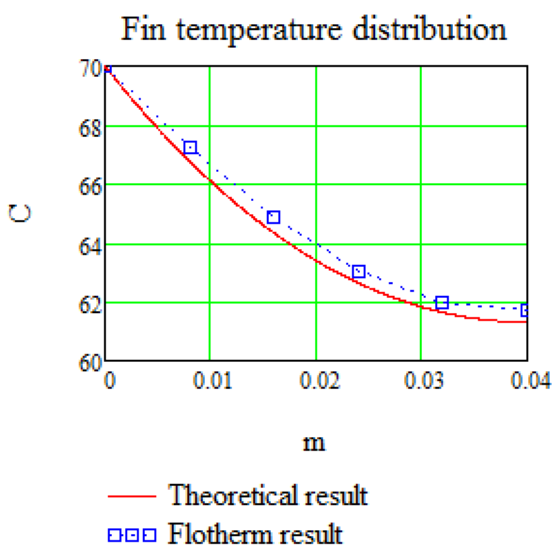
Material: AL 6063 $k = 204 \dfrac{W}{m \cdot C}$

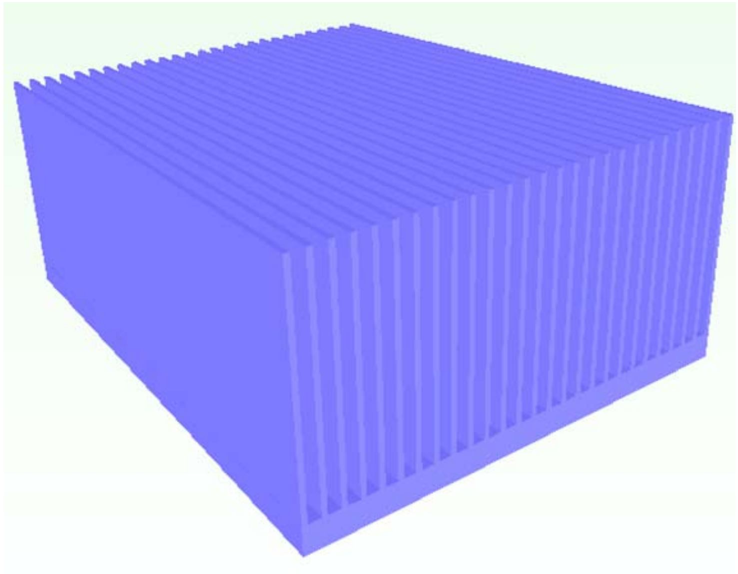
h (3.5m/s) : $h = 40 \dfrac{W}{m^2 C}$

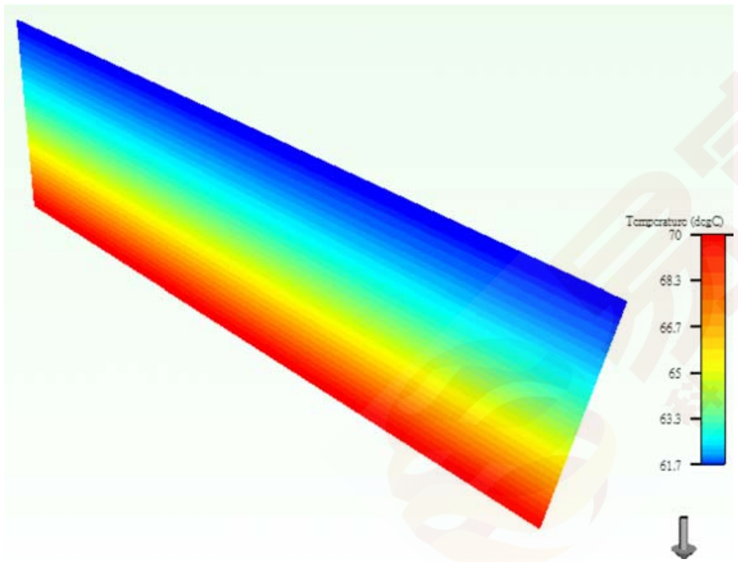
$$M = \sqrt{\frac{2 \cdot h}{k \cdot t}} = 19.803 \frac{1}{m}$$

Bourdary condition: $T_b = 70C$ $\dfrac{d}{dx} T_e = 0$ $T_a = 35C$

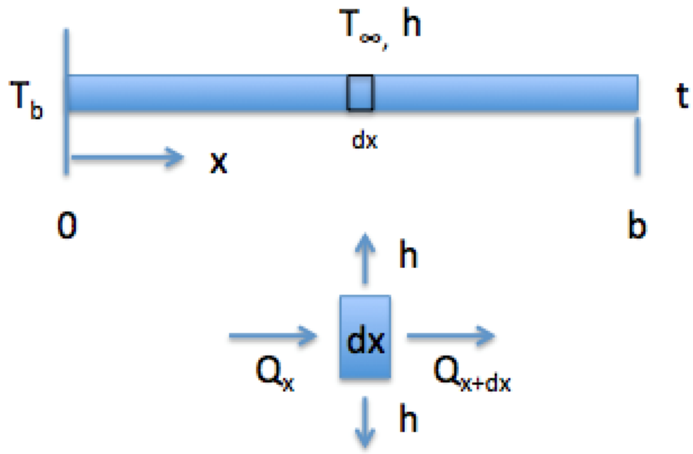
$$T(x) = \left(T_b - T_a\right) \cdot \frac{\cosh[M \cdot (b - x)]}{\cosh(M \cdot b)} + T_a$$







FIN EQUATION TRANSIENT ANALYSIS



$Q_{in} - Q_{out} + Q_g = Q_{st}$

$$Q_x - Q_{x+dx} - 2h\,dx\,L(T - T_\infty) = \rho C_p\,dx\,L \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} - \frac{2h}{kt}(T - T_\infty) = \frac{\rho c_p}{k}\frac{\partial T}{\partial t}$$

Let $m = \sqrt{\frac{2h}{kt}}$

$$\frac{\partial^2 T}{\partial x^2} - m^2 (T - T_\infty) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

BCs

$T(0) = T_b \quad T'(b) = 0$

To Non-dimensionalization

Let $\theta = \frac{T - T_b}{T_b - T_\infty}$, $X = \frac{x}{b}$, $M = \sqrt{\frac{2h}{kt}}\,b$, $\tau = \frac{t\alpha}{b^2}$

$$\frac{\partial^2 T}{\partial x^2} - m^2 (T - T_\infty) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{\Delta T}{b^2}\frac{\partial^2 \theta}{\partial x^2} - m^2 \Delta T \theta = \frac{1}{\alpha}\frac{\Delta T}{b}\frac{\alpha}{b^2}\frac{\partial \theta}{\partial t}$$

$$\Rightarrow \frac{1}{b^2}\frac{\partial^2 \theta}{\partial x^2} - m^2 \theta = \frac{1}{\alpha}\frac{\alpha}{b^2}\frac{\partial \theta}{\partial t}$$

$$\Rightarrow \frac{\partial^2 \theta}{\partial x^2} - m^2 b^2 \theta = \frac{\partial \theta}{\partial t}$$

$$\Rightarrow \frac{\partial^2 \theta}{\partial x^2} - M^2 \theta = \frac{\partial \theta}{\partial t}$$

BCs

$\theta(0) = 1 \quad \theta'(1) = 0$

SEPARATION OF VARIABLES TO SOLVE PDE

$$\frac{\partial^2\theta}{\partial X^2} - M^2\theta = \frac{\partial\theta}{\partial\tau}$$

BCs

$\theta(0)=1 \quad \theta'(1)=0$

Separation variables for analitical solution

Let $\theta=\Phi T$

$$\frac{\partial^2\theta}{\partial x^2} - M^2\theta = \frac{\partial\theta}{\partial\tau}$$

$$\Rightarrow \Phi''T - M^2\Phi T = \Phi T' \Rightarrow \frac{\Phi''T}{\Phi T} - M^2\frac{\Phi T}{\Phi T} = \frac{\Phi T'}{\Phi T} \Rightarrow \frac{\Phi''}{\Phi} = \frac{T'}{T} + M^2 = -\lambda^2$$

$$\Rightarrow \frac{\Phi''}{\Phi} = -\lambda^2$$

$\Rightarrow \Phi'' + \lambda^2\Phi = 0$---(1)

$\Rightarrow \Phi = A\ Cos(\lambda X) + B\ Sin(\lambda X)$

From BCs $\Phi(0)=0 \Rightarrow \Phi = A\ Sin(\lambda X)$

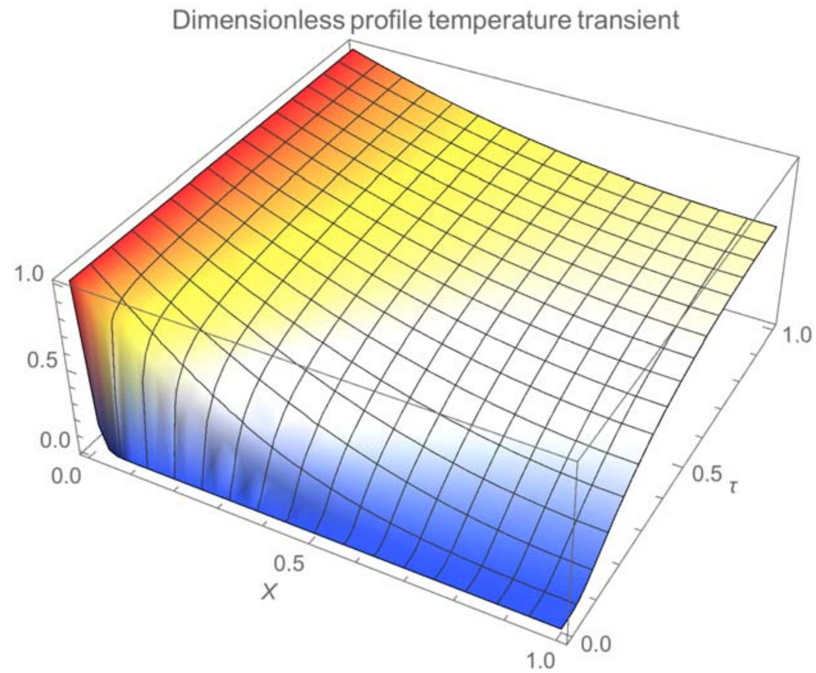
From BCs $X'(1)=0 \Rightarrow X'(1)=\lambda A Cos(\lambda * 1)=0$, $\lambda = \left(n+\frac{1}{2}\right)\pi$, n=0,1,2,3 ......

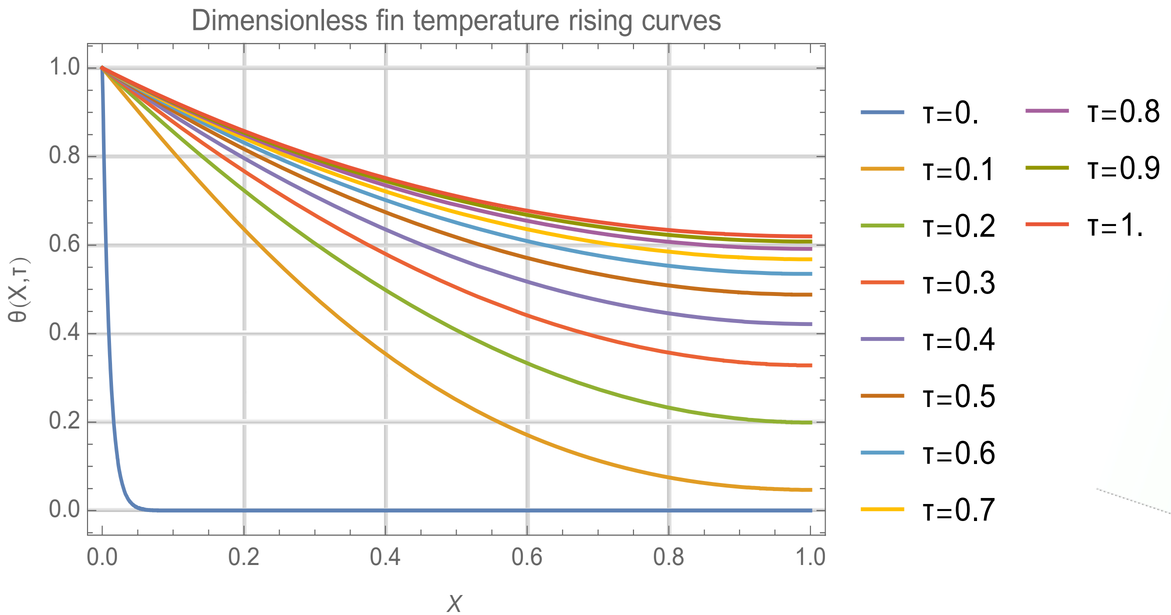
$\Rightarrow T' + M^2 T + \lambda^2 T = 0$ ---(2)

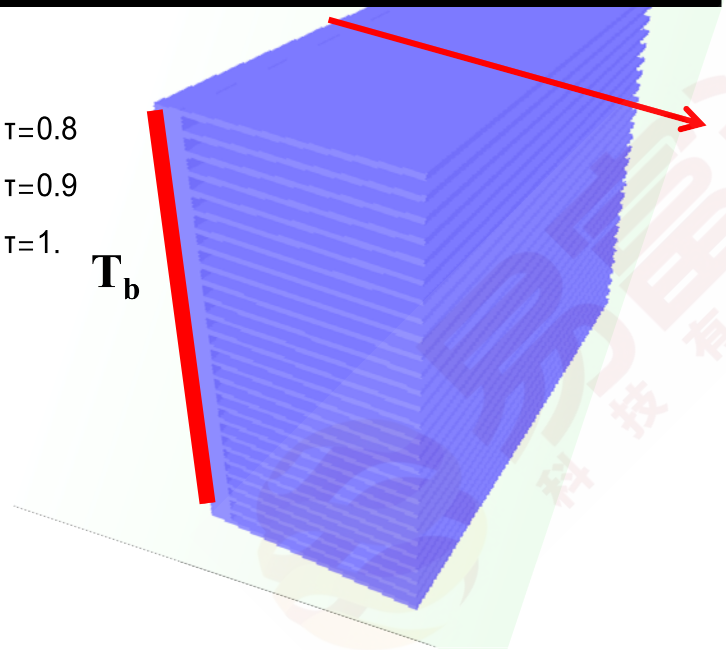
$\Rightarrow T(\tau) = e^{-(m^2+\lambda^2)\tau}$

$\theta(X,\tau) = \sum_{n=0}^{\infty} A_n\ Sin(\lambda_n X) e^{-(m^2+\lambda_n^2)\tau}$

ANALYTICAL SOLUTION VALIDATION







DIFFUSIVITY OF 6063

Thermal diffusivity of selected materials and substances[10]

Material	Thermal diffusivity (m^2/s)	Thermal diffusivity (mm^2/s)
Pyrolytic graphite, parallel to layers	1.22×10^{-3}	1220
Silver, pure (99.9%)	1.6563×10^{-4}	165.63
Gold	1.27×10^{-4} [11]	127
Copper at 25 °C	1.11×10^{-4} [12]	111
Aluminium	9.7×10^{-5} [11]	97
Al-10Si-Mn-Mg (Silafont 36) at 20 °C	74.2×10^{-6} [13]	74.2
Aluminium 6061-T6 Alloy	6.4×10^{-5} [11]	64
Al-5Mg-2Si-Mn (Magsimal-59) at 20 °C	44.0×10^{-6} [14]	44.0
Steel, AISI 1010 (0.1% carbon)	1.88×10^{-5} [15]	18.8

AL6063 α=80mm^2/sec

ANALYTICAL SOLUTION VALIDATION

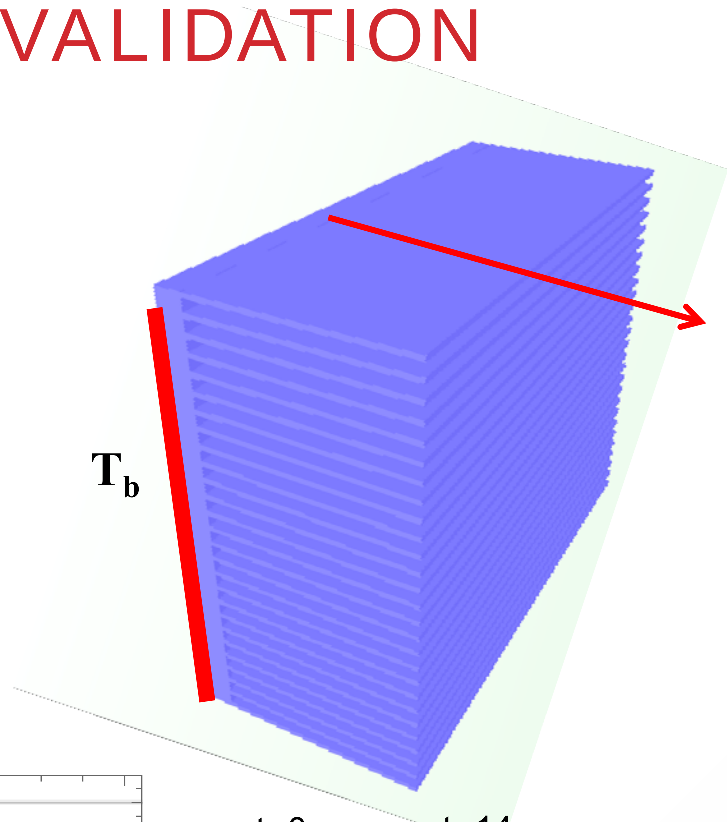
Dimension : $t = 1\text{mm} \quad L = 108\text{mm} \quad b = 40\text{mm}$

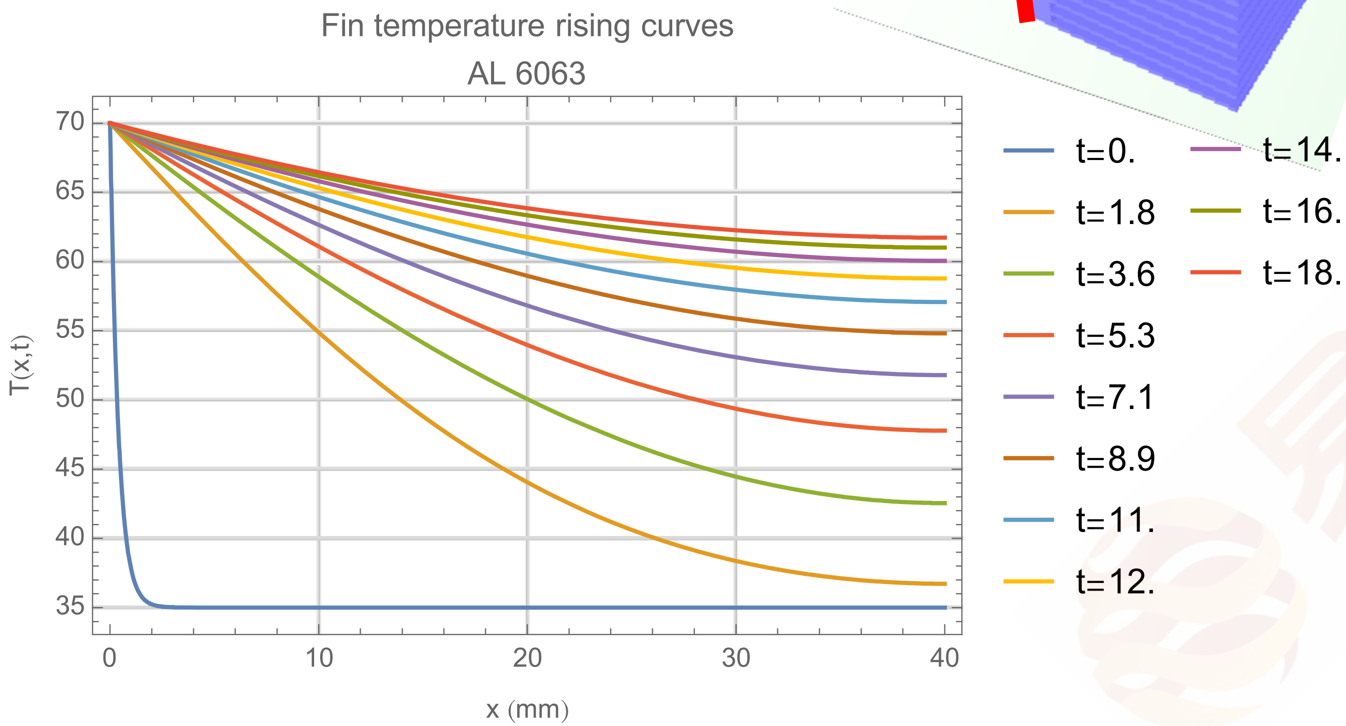
Material: AL 6063 $\quad k = 204 \dfrac{W}{m \cdot C}$

h (3.5m/s) : $h = 40 \dfrac{W}{m^2 C}$

$$M = \sqrt{\frac{2 \cdot h}{k \cdot t}} = 19.803 \frac{1}{m} \qquad M \cdot b = 0.792 \qquad \alpha = 80 \frac{mm^2}{sec}$$

Bourdary condition: $\quad T_b = 70C \quad \dfrac{d}{dx} T_e = 0 \quad T_a = 35C$





ANALYTICAL SOLUTION VALIDATION

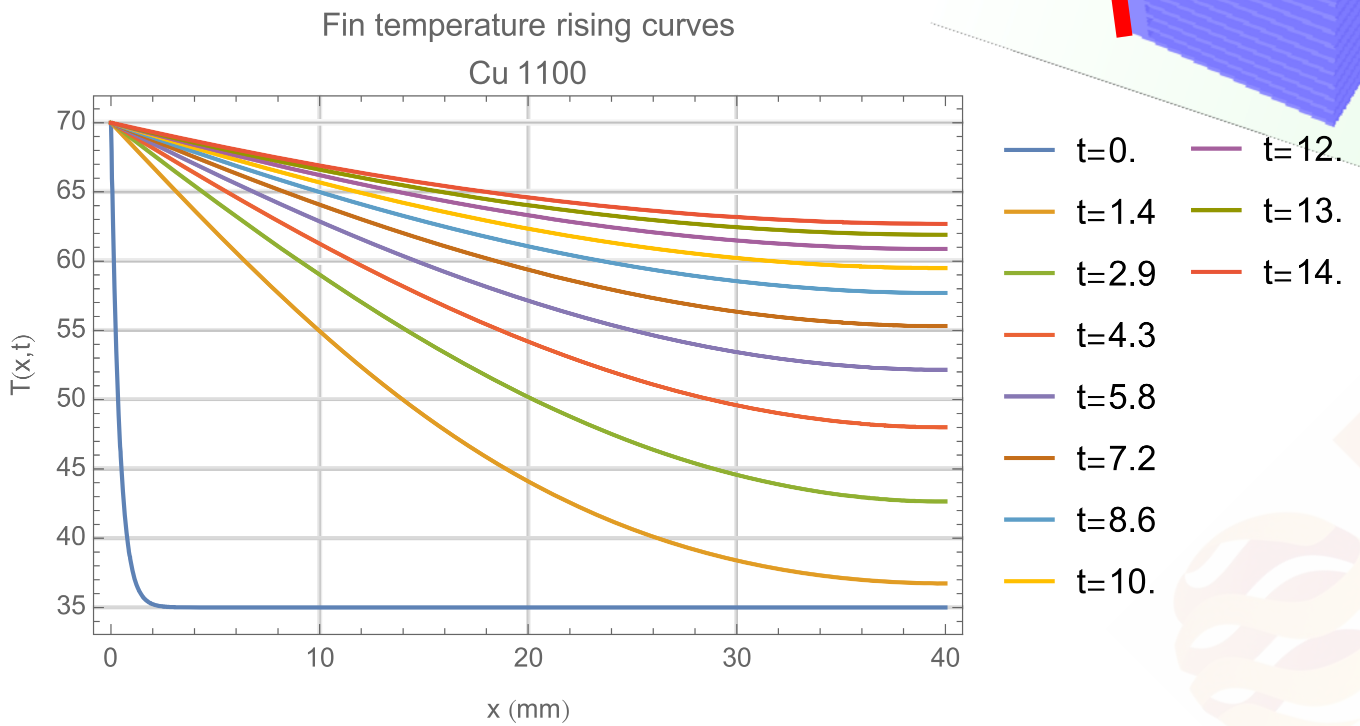
Dimension : $t = 1\text{mm}$ $L = 108\text{mm}$ $b = 40\text{mm}$

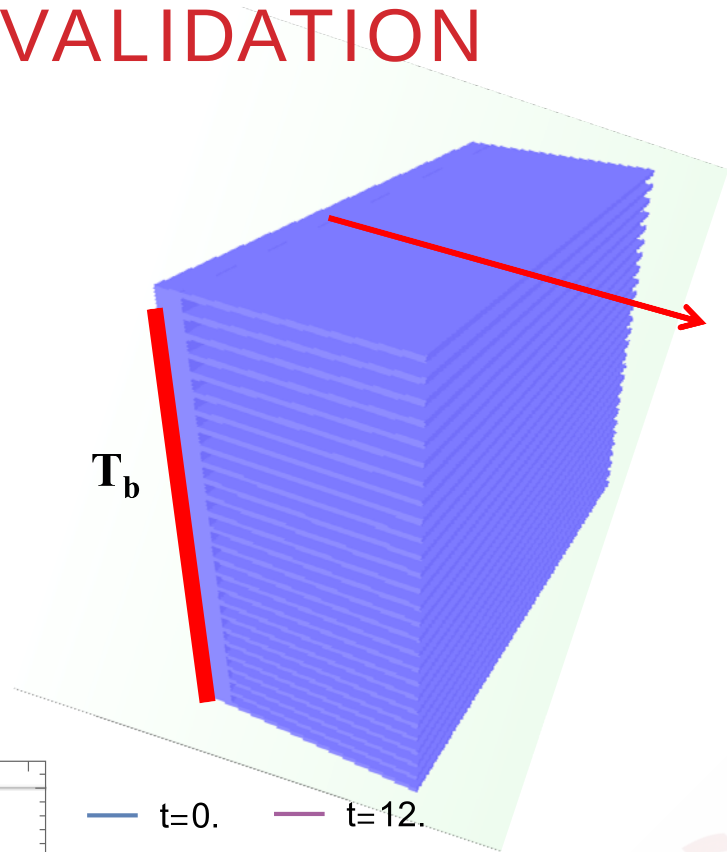
Material: Cu 1100 $k = 396 \dfrac{W}{m \cdot C}$

h (3.5m/s) : $h = 40 \dfrac{W}{m^2 C}$

$M = \sqrt{\dfrac{2 \cdot h}{k \cdot t}} = 14.213 \dfrac{1}{m}$ $M \cdot b = 0.569$ $\alpha = 111 \dfrac{mm^2}{sec}$

Bourdary condition: $T_b = 70C$ $\dfrac{d}{dx} T_e = 0$ $T_a = 35C$





FLOTHERM TRANSIENT MODEL

Spec.

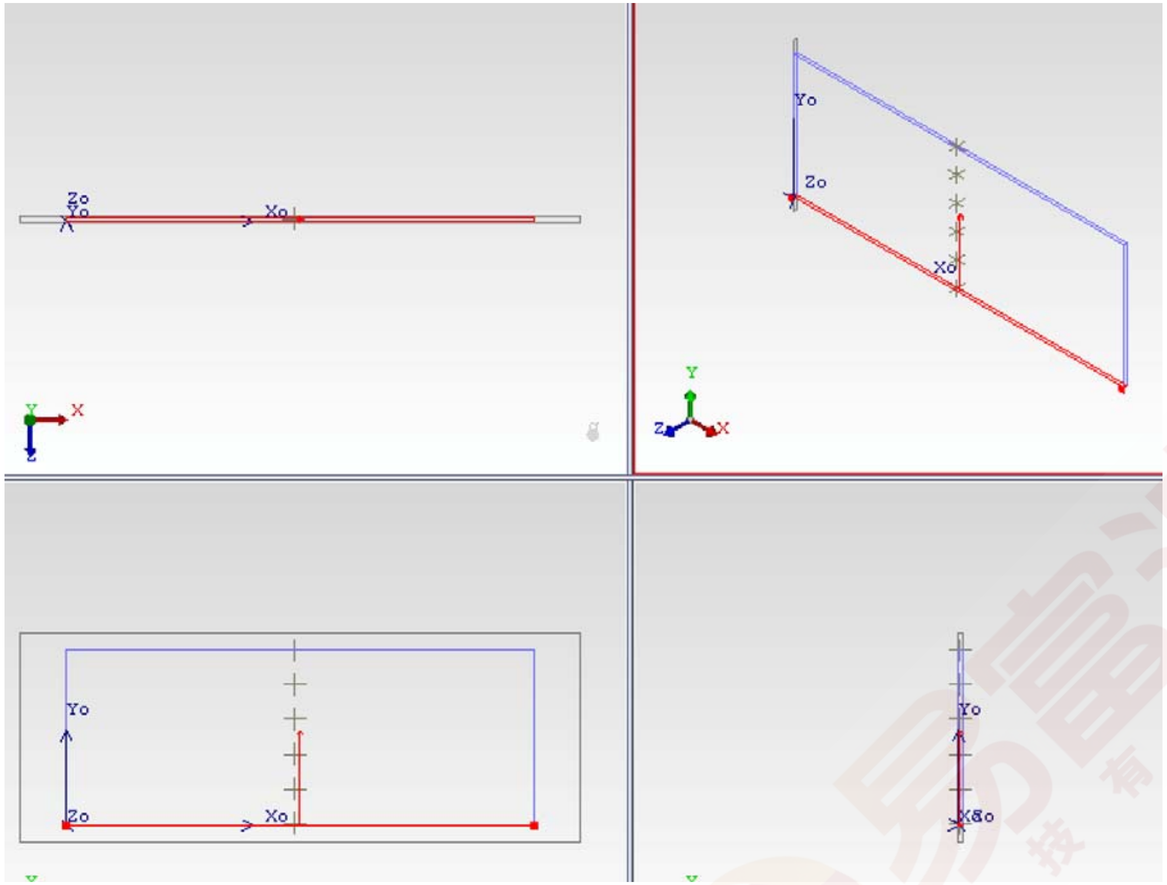
Material :

AL 6063/ K 200W/m C

Ambient: 35C

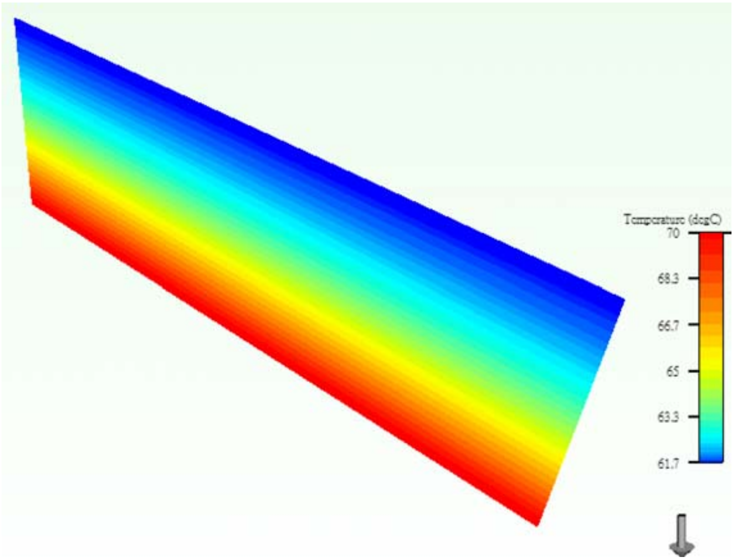
Base temp: 70C

h: 40W/m^2 C

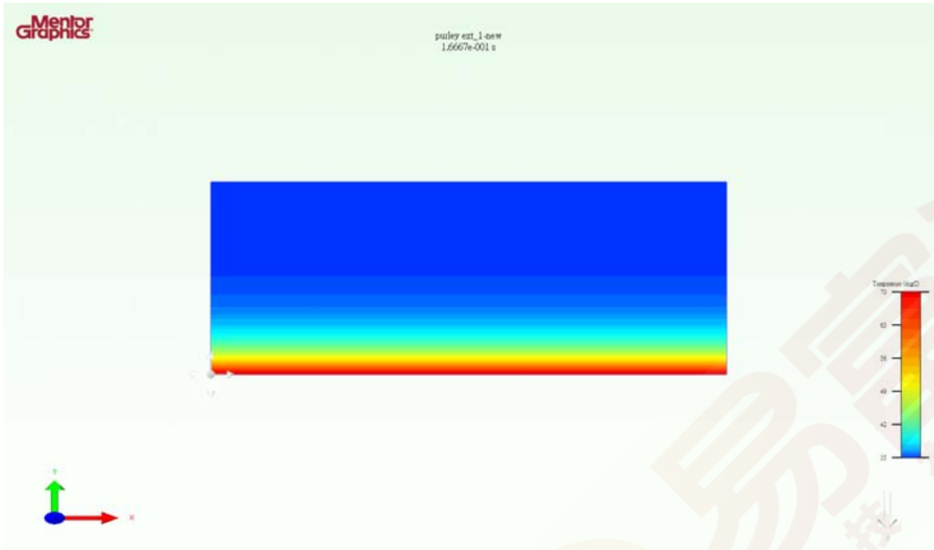


FLOTHERM STEADY & TRANSIENT MODEL

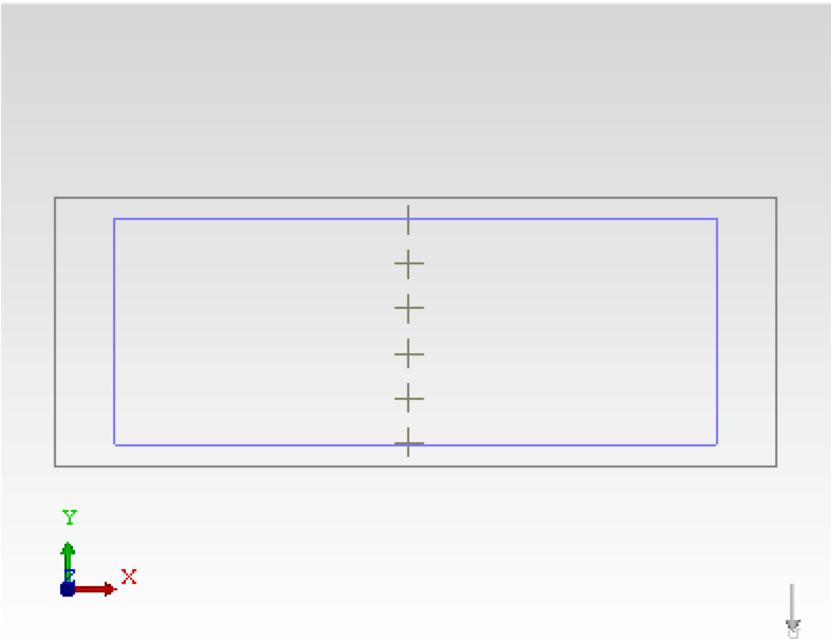
Steady state result

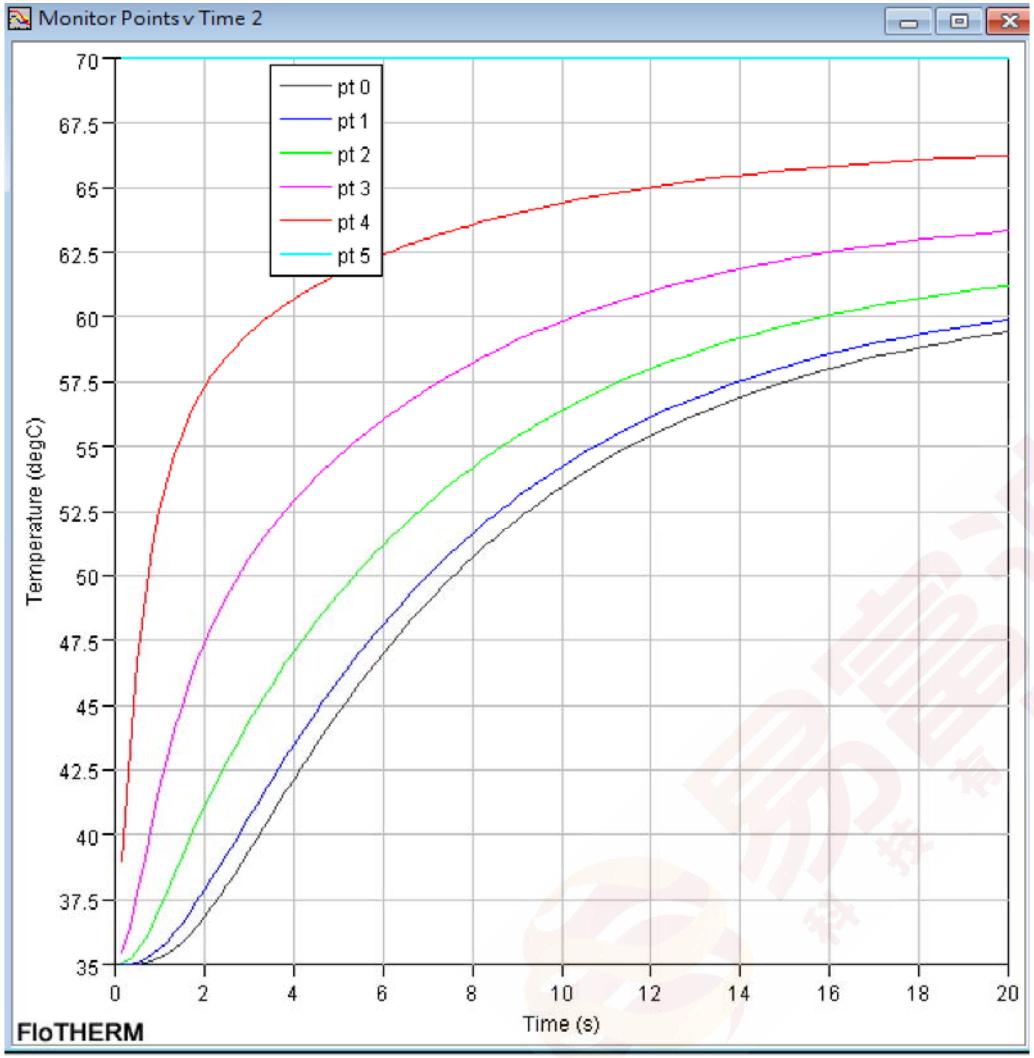


Transient state result

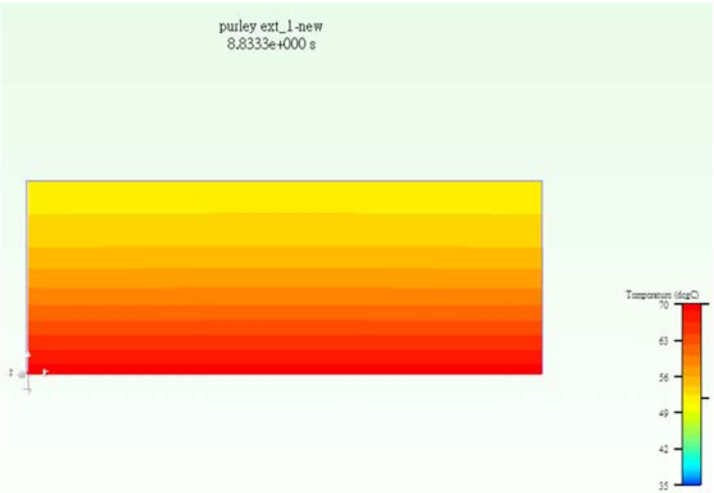


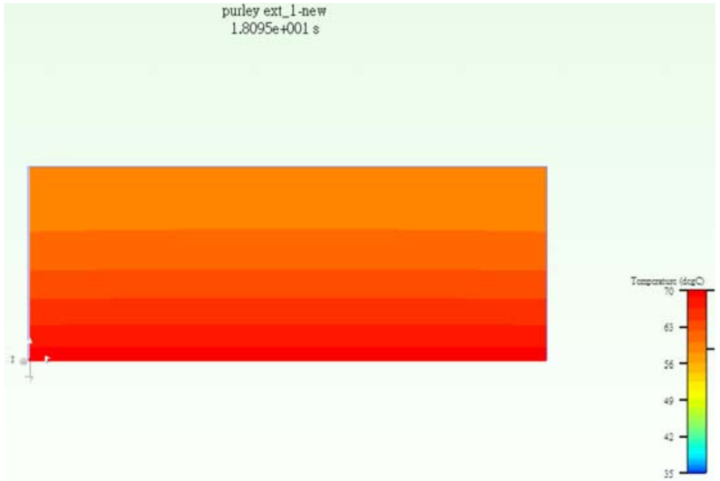
FLOTHERM MONITOR POINTS

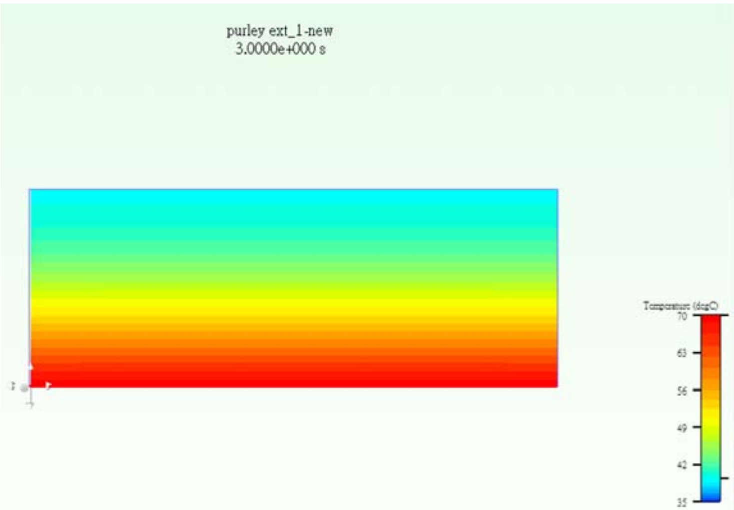


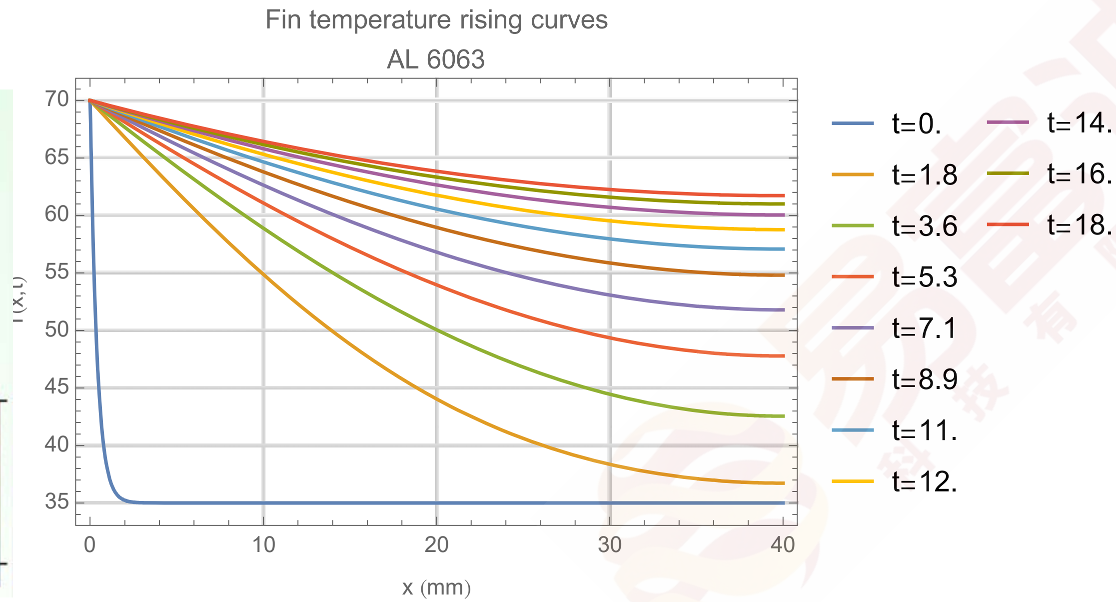


THEORETICAL RESULT V.S. FLOTHERM RESULT









CONCLUSION

- **Transient analysis is a trend for evaluating mobile fanless devices and test timing duration of heat sink performance test platform.**
- **Steady state solution is easy to validate for numerical simulation for only space variable (x,y,z) involved.**
- **Transient state solution is difficult to validate for time variable t involved other than space domain**
- **After using theoretical analysis to validate Flotherm transient module, it is found the trend is in agreement with theoretical result with good accuracy!!.**
- **Future work – validate by 2D theoretical solution.**

THANKS FOR YOUR ATTENTION.